COURSE OUTLINE BRIEFS

DEPARTMENT OF MATHEMATICS

SARGODHA UNIVERSITY
Pathway to Progress

FACULTY OF SCIENCES

OVERVIEW

Mathematics is the engine behind Science in the 21st Century. It has both an inherent logic and beauty while also providing the structure and models from which physicists, chemists, biologists, medics, engineers, economists and social scientists build an understanding of our world and construct the tools to improve our lives.

The Department of Mathematics at University of Sargodha was functional from the time of establishment of Government College Sargodha in 1929. However, post-graduate program has been started in 1985. The Department of Mathematics is committed to make important contributions to undergraduate and graduate teaching, research and innovation across the full range of mathematical disciplines (Pure and Applied Mathematics, Computational Mathematics). In addition to more traditional graduate programs, the Department of Mathematics offers MPhil and Ph.D. programs with a variety of research areas e.g., Computer Aided Geometric Designs, Fixed point theory, Fluid Mechanics, Mathematical inequalities, Numerical Analysis, Special functions and Theory of time scales.

Department of Mathematics has a fully equipped computer laboratory. Faculty members of the Department remain very active in outreach and enrichment to promote teaching and research activities. The Department organizes national and international conferences every year in which faculty, speakers (well-known National and International Scholars) and research students present topics of current research or historical interest.

Department of Mathematics has established educational collaboration with well-known National and International institutions to facilitate faculty members and students with latest research and teaching environment.

Academic Programs Offered

1. BS Mathematics
2. M. Sc. Mathematics
3. M. Phil. Mathematics
4. Ph. D. Mathematics

BS Mathematics

Eligibility: Intermediate or equivalent with Mathematics (at least 45% marks in Intermediate & 50% marks in Mathematics).
Duration: 04 Years Program (08 Semesters)
Degree Requirements: Minimum 124 Credit Hours

Semester-1

Course Code	Course Title	Credit Hours
MATH-5101	Calculus-I	3(3+0)
MATH-5102	Elements of Set Theory and Mathematical Logic	3(3+0)
PHYS-5161	Physics-I	4(3+1)
URCE-5101	Grammar	3(3+0)
URCP-5106	Pakistan Studies	2(2+0)
URCI-5109	Introduction to Information and Communication Technologies	3(3+0)

Semester-2

MATH-5103	Calculus-II	3(3+0)
MATH-5104	Statistics	3(3+0)
PHYS-5162	Physics-II	4(3+1)
URCE-5102	Language Comprehension & Presentation Skills	3(3+0)
URCI-5105	Islamic Studies	2(2+0)
MATH-5105	Programming Languages for Mathematicains	3(2+1)
No-Credit Course		
URCC-5110	Citizenship Education and Community Engagement	3(1+2)

Semester-3

MATH-5106	Calculus-III	3(3+0)
MATH-5107	Algebra-I	3(3+0)
PHYS-5163	Physics-III	4(3+1)
URCE-5103	Academic Writing	3(3+0)
MATH-5108	Probability Theory	3(3+0)

Semester-4

MATH-5109	Vector Analysis & Mechanics	3(3+0)
MATH-5110	Linear Algebra	3(3+0)
PHYS-5164	Physics-IV	4(3+1)
MATH-5111	Discrete Mathematics	3(3+0)

MATH-5112/ MATH-5113/ ECON-5118	Spanish/French/Mathematical Economics	3(3+0)

Semester-5

MATH-6112	Topology	3(3+0)
MATH-6113	Differential Geometry	3(3+0)
MATH-6114	Ordinary Differential Equations	3(3+0)
MATH-6115	Real Analysis-I	3(3+0)
MATH-6116	Algebra-II	3(3+0)

Semester-6

MATH-6117	Classical Mechanics	3(3+0)
MATH-6118	Mathematical Methods	3(3+0)
MATH-6119	Complex Analysis	3(3+0)
MATH-6120	Functional Analysis	3(3+0)
MATH-6121	Real Analysis-II	3(3+0)

Semester-7

MATH-6122	Numerical Analysis-I	3(3+0)
MATH-6123	Number Theory	3(3+0)
MATH-6124	Partial Differential Equations	3(3+0)
MATH-61xx	Elective-I*	3(3+0)
MATH-61xx	Elective-II*	3(3+0)

Semester-8

MATH-6125	Numerical Analysis-II	3(3+0)
MATH-6126	Integral Equations	3(3+0)
MATH-61xx	Project / Course**	3(3+0)
MATH-61xx	Elective-III*	3(3+0)
MATH-61xx	Elective-IV*	3(3+0)

Total Numbers of Credit Hours=128

* These four courses are optional & can be selected either from list A or B but cannot be mixed from both or any two courses can be selected from list C.

** In lieu of dissertation a course can be selected from list C.

*** Any other language can be added according to availability of resources.

Note: These courses will be offered by the department from the lists of concentration elective courses & free elective courses as per availability of the resources.

List of Concentration Elective Courses

A student must satisfactorily complete 12 credit hours of any one of the following concentration groups of Elective Courses namely, Pure or Applied Mathematics.

MATH-6127	Advanced Group Theory-I	3(3+0)

MATH-6128	Advanced Group Theory-II	3(3+0)
MATH-6129	Modern Algebra-I	3(3+0)
MATH-6130	Modern Algebra-II	3(3+0)
MATH-6131	Algebraic Topology-I	3(3+0)
MATH-6132	Algebraic Topology-II	3(3+0)
MATH-6133	Advanced Functional Analysis	3(3+0)
MATH-6134	Theory of Modules	3(3+0)

List A: Concentration Elective Courses in Pure Mathematics

List B: Concentration Elective Courses in Applied Mathematics

MATH-6135	Astronomy-I	3(3+0)
MATH-6136	Astronomy-II	3(3+0)
MATH-6137	Electromagnetism-I	3(3+0)
MATH-6138	Electromagnetism-II	3(3+0)
MATH-6139	Fluid Mechanics-I	3(3+0)
MATH-6140	Fluid Mechanics-II	3(3+0)
MATH-6141	Operations Research-I	3(3+0)
MATH-6142	Operations Research-II	3(3+0)
MATH-6143	Quantum Mechanics-I	3(3+0)
MATH-6144	Quantum Mechanics-II	3(3+0)
MATH-6145	Analytical Dynamics	3(3+0)
MATH-6146	Special Relativity	3(3+0)

List C: List of Free Elective Courses

A student must also satisfactorily complete 06 credits of any one of the following free Elective courses in Applied & Pure Mathematics.

MATH-6147	Numerical Solution of Partial differential equations	3(3+0)
MATH-6148	Elasticity Theory	3(3+0)
MATH-6149	History of Mathematics	3(3+0)
MATH-6150	Heat Transfer	3(3+0)
MATH-6151	Bio-Mathematics	3(3+0)
MATH-6152	Theory of Automata	3(3+0)
MATH-6153	Measure Theory	3(3+0)
MATH-6154	Special Functions	3(3+0)
MATH-6155	Theory of Splines-I	3(3+0)
MATH-6156	Theory of Splines-II	3(3+0)
MATH-6157	Methods of Optimization-I	3(3+0)
MATH-6158	Methods of Optimization-II	3(3+0)
MATH-6159	Control Theory	3(3+0)
MATH-6160	Applied Matrix Theory	3(3+0)

Note: Other elective courses can be offered according to availability of resources.

M. Sc. Mathematics

Eligibility: At least 45% marks in B.Sc. with Mathematics A & B courses & 50% marks in Mathematics A & B courses.
Duration: 02 Years Program (04 Semesters)
Degree Requirements: Minimum 66 credit hours

Semester-1

MATH-6201	Real Analysis-I	3 (3+0)
MATH-6202	Abstract Algebra	3 (3+0)
MATH-6203	Topology	3 (3+0)
MATH-6204	Vector & Tensor Analysis	3 (3+0)
MATH-6205	Set Theory and ODEs	3 (3+0)
URCI-5109	Introduction to Information and Communication Technologies	3 (2+1)

Semester-2

MATH-6206	Real Analysis-II	3 (3+0)
MATH-6207	Linear Algebra	3 (3+0)
MATH-6208	Functional Analysis	3 (3+0)
MATH-6209	Mechanics	3 (3+0)
MATH-6210	Complex Analysis	3 (3+0)
MATH-6211	Computer Programming with C++	3 (2+1)
No-Credit Course		
URCC-5110	Citizenship Education and Community Engagement	3(1+2)

Semester-3

MATH-6212	Differential Geometry	3 (3+0)
MATH-6213	Partial Differential Equations	3 (3+0)
MATH-6214	Numerical Analysis-I	3 (3+0)
Pure Mathematics Group		
MATH-62XX	Elective-I*	3 (3+0)
MATH- 62XX	Elective-II*	3 (3+0)
Applied Mathematics Group		
MATH- 62XX	Elective-I**	3 (3+0)
MATH- 62XX	Elective-II**	3 (3+0)
Computational Mathematics Group		
MATH- 62XX	Elective-I***	3 (3+0)
MATH- 62XX	Elective-II***	3 (3+0)

Semester-4

MATH-6215	Probability Theory	3 (3+0)
MATH-6216	Integral Equations	3 (3+0)
MATH-6217	Numerical Analysis-II	3 (3+0)
MATH-6218	History of Mathematics	2 (2+0)
Pure Mathematics Group		
MATH-62XX	Elective-I*	3 (3+0)

MATH- 62XX	Elective-II*	3 (3+0)
Applied Mathematics Group		
MATH- 62XX	Elective-I**	3 (3+0)
MATH- 62XX	Elective-II**	3 (3+0)
Computational Mathematics Group		
MATH- 62XX	Elective-I***	3 (3+0)
MATH- 62XX	Elective-II***	3 (3+0)

Common Elective Courses

MATH-6219	Special Functions	3 (3+0)
MATH-6220	Algorithms and Computer Programming	3 (2+1)
MATH-6221	Advanced Programming for Scientific Computing	3 (2+1)

List A: Courses of Pure Mathematics

MATH-6222	Analytic Number Theory	3 (3+0)
MATH-6223	Algebraic Number Theory	3 (3+0)
MATH-6224	Advanced Group Theory –I	3 (3+0)
MATH-6225	Advanced Group Theory –II	3 (3+0)
MATH-6226	Algebraic Topology-I	3 (3+0)
MATH-6227	Algebraic Topology-II	3 (3+0)
MATH-6228	Category Theory –I	3 (3+0)
MATH-6229	Category Theory–II	3 (3+0)
MATH-6230	Rings & Fields	3 (3+0)
MATH-6231	Theory of Modules	3 (3+0)
MATH-6232	Lie Algebra	3 (3+0)
MATH-6233	Advanced Functional Analysis	3 (3+0)
MATH-6234	Galois Theory	3 (3+0)
MATH-6235	Measure Theory	3 (3+0)

List B: Courses of Applied Mathematics

MATH-6236	Fluid Mechanics-I	3 (3+0)
MATH-6237	Fluid Mechanics-II	3 (3+0)
MATH-6238	Quantum Mechanics-I	3 (3+0)
MATH-6239	Quantum Mechanics-II	3 (3+0)
MATH-6240	Electromagnetic Theory –I	3 (3+0)
MATH-6241	Electromagnetic Theory –II	3 (3+0)
MATH-6242	Special Relativity	3 (3+0)
MATH-6243	Elasticity Theory	3 (3+0)
MATH-6244	Analytical Dynamics	3 (3+0)
MATH-6245	Astronomy-I	3 (3+0)
MATH-6246	Astronomy-II	3 (3+0)

List C: Courses of Computational Mathematics

MATH-6247	Operations Research-I	3 (3+0)
MATH-6248	Operations Research-II	3 (3+0)

MATH-6249	Methods of Optimization-I	3 (3+0)
MATH-6250	Methods of Optimization-II	3 (3+0)
MATH-6251	Theory of Splines-I	3 (3+0)
MATH-6252	Theory of Splines-II	3 (3+0)
MATH-6253	Graph Theory	3 (3+0)
MATH-6254	Theory of Automata	3 (3+0)
MATH-6255	Control Theory	3 (3+0)
MATH-6256	Applied Matrix Theory	3 (3+0)
MATH-6257	Finite Element Analysis	3 (3+0)

Cumulative Credits= 68

***** These courses are optional & can be selected from list A or common electives.

****** These courses are optional & can be selected from list B or common electives.

*******These courses are optional & can be selected from list C or common electives.

M. Phil. Mathematics

Eligibility: M. Sc./BS 4-Year or equivalent (16 years of Education) in the relevant field or equivalent degree from HEC recognized institution with at least second Division or CGPA 2.00 out of 4.00.
Duration: 02 Years Program (04 Semesters)
Degree Requirements: Minimum 30 Credit Hours

Semester-1

MATH-71xx	Elective-I	3(3+0)
MATH-71xx	Elective-II	3(3+0)
MATH-71xx	Elective-III	3(3+0)
MATH-71xx	Elective-IV	3(3+0)

Semester-2

MATH-71xx	Elective-I	3(3+0)
MATH-71xx	Elective-II	3(3+0)
MATH-71xx	Elective-III	3(3+0)
MATH-71xx	Elective-IV	3(3+0)

Semester 3-4

M. Phil. Thesis	6(6+0)

List of elective courses for M. Phil. Mathematics

MATH-7101	Representation Theory-I	3(3+0)
MATH-7102	Semigroup Theory	3(3+0)
MATH-7103	Advanced Ring Theory-I	3(3+0)
MATH-7104	Theory of Group Actions	3(3+0)
MATH-7105	Graph Theory	3(3+0)
MATH-7106	Topological Vector Spaces	3(3+0)
MATH-7107	Advanced Complex Analysis-I	3(3+0)
MATH-7108	Fixed Point Theory	3(3+0)
MATH-7109	Approximation Theory	3(3+0)
MATH-7110	Topological Algebras	3(3+0)
MATH-7111	Commutative Algebra-I	3(3+0)
MATH-7112	Theory of Semirings	3(3+0)
MATH-7113	Partial Differential Equations	3(3+0)
MATH-7114	Computer Aided Geometric Design	3(3+0)
MATH-7115	Magnetohydrodynamics-I	3(3+0)
MATH-7116	Electrodynamics-I	3(3+0)
MATH-7117	Advanced Fluid Mechanics	3(3+0)
MATH-7118	Advanced Analytical Dynamics-I	3(3+0)
MATH-7119	Elastodynamics-I	3(3+0)
MATH-7120	General Relativity	3(3+0)
MATH-7121	Numerical Solutions of Ordinary Differential Equation	3(3+0)
MATH-7122	Advanced Heat Transfer	3(3+0)
MATH-7123	Introduction to Subdivision Scheme	3(3+0)

MATH-7124	Nilpotent & Solvable Groups	3(3+0)
MATH-7125	Convex Analysis	3(3+0)
MATH-7126	Representation Theory-II	3(3+0)
MATH-7127	Advanced Ring Theory-II	3(3+0)
MATH-7128	Theory of Group Graphs	3(3+0)
MATH-7129	Advanced Complex Analysis-II	3(3+0)
MATH-7130	Variational Inequalities	3(3+0)
MATH-7131	Field Extensions & Galois Theory	3(3+0)
MATH-7132	Commutative Algebra-II	3(3+0)
MATH-7133	Commutative Semigroup Rings	3(3+0)
MATH-7134	Cosmology	3(3+0)
MATH-7135	Magnetohydrodynamics-II	3(3+0)
MATH-7136	Electrodynamics-II	3(3+0)
MATH-7137	Mathematical Techniques for Boundary Value Problems	3(3+0)
MATH-7138	Advanced Analytical Dynamics-II	3(3+0)
MATH-7139	Elastodynamics-II	3(3+0)
MATH-7140	Design Theory	3(3+0)
MATH-7141	Acoustics	3(3+0)
MATH-7142	Combinatorics	3(3+0)
MATH-7143	Theory of Majorization	3(3+0)
MATH-7144	Inequalities Involving Convex Functions	3(3+0)
MATH-7145	Harmonic Analysis	3(3+0)
MATH-7146	Research Methodology	3(3+0)
MATH-7147	Integral Transform	3(3+0)
MATH-7148	Advanced Numerical Analysis	3(3+0)
MATH-7149	Generalized Special Functions	3(3+0)
MATH-7150	Scientific Computation	3(3+0)
MATH-7151	Mathematical Modeling-I	3(3+0)
MATH-7152	Mathematical Modeling-II	3(3+0)
MATH-7153	Computer Graphics	3(3+0)
MATH-7154	Dynamic Inequalities on Time Scales	3(3+0)
MATH-7155	Set-Valued Analysis	3(3+0)
MATH-7156	Fractional Calculus	3(3+0)
MATH-7157	Perturbation Methods-I	3(3+0)
MATH-7158	Perturbation Methods-II	3(3+0)
MATH-7159	Viscous Fluids-I	3(3+0)
MATH-7160	Viscous Fluids-II	3(3+0)
MATH-7161	Fuzzy Algebra	3(3+0)
MATH-7162	Minimal Surfaces	3(3+0)
MATH-7163	Riemannian Geometry	3(3+0)

Ph. D. Mathematics

Eligibility: M. Phil./MS (18 years Education) with at least 3.00 CGPA out of 4.00 in semester or 1st division in Annual System.
Duration: 03 Years Program (06 Semesters)
Degree Requirements: Minimum 24 Credit Hours

Semester-1

MATH-81xx	Elective-I	3(3+0)
MATH-81xx	Elective-II	3(3+0)
MATH-81xx	Elective-III	3(3+0)

Semester-2

MATH-81xx	Elective-I	3(3+0)
MATH-81xx	Elective-II	3(3+0)
MATH-81xx	Elective-III	3(3+0)

Semester 3-6

Ph. D. Thesis

Note:

1. MS/M. Phil students will have to pass the 24 credit hours courses, which can be opted from the approved list of M. Phil. courses.

2. A Ph. D. student will have to complete 18 credit hours course work, which can be opted from the approved list of Ph. D. courses.

3. Department can offer any course from the list of approved M. Phil./Ph. D. courses on the availability of resources.

4. The supervisor may recommend a Ph. D. student opt courses of his/her relevant field from approved courses of M. Phil./Ph. D. offered by Department of Physics or Department of Computer Science & Information Techonology to fulfill his/her Ph. D. coursework condition.

5. The supervisor may recommend a Ph. D. student to opt courses of M. Phil to fulfill his/her Ph. D. coursework condition if he/she didn’t study these courses during his/her M. Phil & vice versa.

List of elective courses for Ph.D. Mathematics

MATH-8101	Lie Algebras	3(3+0)
MATH-8102	Numerical Solutions of Integral Equations	3(3+0)
MATH-8103	Multiresolution Analysis in Geometric Modeling	3(3+0)
MATH-8104	Advanced Graph Theory	3(3+0)
MATH-8105	Strict Convexity	3(3+0)
MATH-8106	Applications of Inequalities	3(3+0)
MATH-8107	Propagation of Waves in Different Media	3(3+0)
MATH-8108	Scattering and Diffraction of Elastic Waves	3(3+0)
MATH-8109	Lattice Theory	3(3+0)
MATH-8110	Gravitational Collapse & Black Holes	3(3+0)

MATH-8111	Spectral Theory in Hilbert Spaces- I	3(3+0)
MATH-8112	Spectral Theory in Hilbert spaces – II	3(3+0)
MATH-8113	Multivariate Analysis-I	3(3+0)
MATH-8114	Multivariate Analysis-II	3(3+0)
MATH-8115	Spacetimes Foliations	3(3+0)
MATH-8116	Teleparallel Theory of Gravity	3(3+0)
MATH-8117	Homotopy Theory	3(3+0)
MATH-8118	Symmetries of Spacetimes	3(3+0)
MATH-8119	Convex Analysis and Applications	3(3+0)
MATH-8120	Numerical Solutions of Partial Differential Equations	3(3+0)
MATH-8121	Representation Theory and the Symmetric Groups	3(3+0)
MATH-8122	Non-Newtonian Fluid Mechanics	3(3+0)
MATH-8123	Orthogonal Polynomials	3(3+0)
MATH-8124	Numerical Spline Techniques-I	3(3+0)
MATH-8125	Numerical Spline Techniques-II	3(3+0)
MATH-8126	Computational Geometry	3(3+0)
MATH-8127	Special Functions and Statistical Distribution	3(3+0)
MATH-8128	Fuzzy Analysis	3(3+0)
MATH-8129	Advanced Partial Differential Equation	3(3+0)

BS MATHEMATICS

MATH-5101 **Calculus-I** **3 (3+0)**

Calculus is the mathematical study of continuous change. If quantities are continually changing, we need calculus to study what is going on. Calculus is concerned with comparing quantities which vary in a non-linear way. It is used extensively in science & engineering, since many of the things we are studying (like velocity, acceleration, current in a circuit) do not behave in a simple, linear fashion. Calculus has two major branches, differential calculus (Calculus – I) & integral calculus (Calculus – II); the former concerns instantaneous rates of change, & the slopes of curves, while integral calculus concerns accumulation of quantities, & areas under or between curves. This is the first course of the sequence, Calculus-I, II & III, serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts & skills needed for mathematical manipulation. It focuses on the study of functions of a single variable. Calculus-I is an introduction to differential & integral calculus: the study of change.

Contents

1. Functions & their graphs, Rates of change & tangents to curves
2. Limit of a function & limit laws, the precise definition of a limit
3. One-sided limits, continuity, Limits involving infinity; asymptotes of graphs
4. Differentiation: tangents & derivative at a point, the derivative as a function
5. Differentiation rules, the derivative as a rate of change
6. Derivatives of trigonometric functions, Chain rule, implicit differentiation
7. Related rates, linearization & differentials, higher derivatives
8. Applications of derivatives: extreme values of functions
9. Rolls' theorem, the mean value theorem, Monotonic functions & the first derivative test
10. Convexity, point of inflection & second derivative test, Concavity & curve sketching
11. Applied optimization, Antiderivatives, integration: area & estimating with finite sums
12. Sigma notation & limits of finite sums, definite integral, the fundamental theorem of calculus
13. Indefinite integrals & the substitution method, Substitution & area between curves
14. Applications of definite integrals: volumes using cross-sections

Recommended Texts

1. Thomas, G. B., Weir, M. D., & Hass J. R. (2014). *Thomas'calculus: single variable* (13th ed.). London: Pearson.
2. Stewart, J. (2015). *Calculus* (8th ed.). Boston: Cengage Learning.

Suggested Readings

1. Anton, H., Bivens I. C., & Davis, S. (2016). *Calculus* (11th ed.). New York: Wiley.
2. Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed.). London: Pearson.
3. Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed.). New York: Brooks Cole.

MATH-5102 Elements of Set Theory and Mathematical Logic 3 (3+0)

The main aim of this course is the study of set theory & the concept of mathematical logic. Everything mathematicians do can be reduced to statements about sets, equality & membership which are basics of set theory. This course introduces these basic concepts. The foundational role of set theory & its mathematical development have raised many philosophical questions that have been debated since its inception in the late nineteenth century. The course begins with propositional logic, including two-column proofs and truth table applications, followed by first-order logic, which provides the structure for writing mathematical proofs. Set theory is then introduced and serves as the basis for defining relations, functions, numbers, mathematical induction, ordinals, and cardinals. In particular, mathematicians have shown that virtually all mathematical concepts & results can be formalized within the theory of sets. The course aims at familiarizing the students with cardinals, ordinal numbers, relations, functions, Boolean algebra, fundamentals of propositional & predicate logics.

Contents

1. Set theory: sets, subsets
2. Operations with sets: union, intersection, difference, symmetric difference
3. Cartesian product & disjoint union
4. Functions: graph of a function
5. Composition; injections, surjections, bijections, inverse function
6. Computing cardinals: Cardinality of Cartesian product, union
7. Cardinality of all functions from a set to another set
8. Cardinality of all injective, surjective & bijective functions from a set to another set
9. Infinite sets, finite sets, Countable sets, properties & examples
10. Operations with cardinal numbers. Cantor-Bernstein theorem
11. Relations: equivalence relations
12. Partitions, quotient set; examples
13. Parallelism, similarity of triangles
14. Order relations, min, max, inf, sup; linear order
15. Examples: N, Z, R, P(A). Well ordered sets & induction
16. Inductively ordered sets & Zorn's lemma
17. Mathematical logic: propositional calculus. truth tables
18. Predicate calculus

Recommended Texts

1. Halmos, P. R. (2019). *Native set theory*. New York: Bow Wow Press.
2. Lipschuts, S. (1998). *Schaum's outline of set theory & related topics* (2nd ed.). New York: McGraw-Hill Education.

Suggested Readings

1. Pinter, C. C. (2014). *A book of set theory*. New York: Dover Publication.
2. O'Leary, M. L. (2015). *A first course in mathematical logic & set theory* (1st ed.). New York: Wiley.
3. Smith, D., Eggen, M., & Andre, R.S. (2014). *A transition to advanced mathematics* (8th ed.). New York: Brooks/Cole.

PHYS- 5161 **Physics-I** **4(3+1)**

This is the first part of a two-semester sequence. This course provides a thorough introduction to the principles and methods of physics for students who have good preparation in physics and mathematics. Emphasis is placed on problem solving and quantitative reasoning. This course covers vector analysis, particle dynamics, fluid dynamics and thermodynamics. This course has a laboratory component. Upon successful completion of this course, students should be able to convert between different units and express a physical quantity in scientific notation using the appropriate number of significant digits, explain the relationships between time, displacement, velocity and constant acceleration, and use algebra to solve kinematic problems in one or two imensions, to analyze and solve dynamic problems using vector addition, Newton's three laws of motion, and resistive forces, to analyze and solve work, energy and power-related problems using appropriate formulas and the conservation of energy principle.

Contents

1. Vector Analysis
2. Particle Dynamics, System of Particles, Circular Motions
3. Rotational Dynamics, Angular Momentum, Collisions, Work Power & Energy, Gravitation
4. Fluid Mechanics
5. Bulk Properties of Matters
6. Waves & Oscillations
7. Harmonic Oscillations

Lab-I

1. Modulus of rigidity by static & dynamic method (Maxwell's needle, Barton's
2. Apparatus)
3. Determination of moment of inertia of a solid/hollow cylinder & a sphere etc.
4. To study the conservation of energy (Hook's Law)
5. To determine the surface tension of water by capillary tube method
6. To determine the value of 'g' by a compound pendulum
7. To study the laws of vibration of stretched string-using sonometer

Recommended Texts

1. Raymond, A., & Jewett, J. W. (2011). *Physics for scientists & engineers with modern physics* (8th ed.). Boston: Cengage Learning.
2. Halliday, D., Resnick, R., & Walker, J. (2008). *Fundamental of physics, extended* (8th ed.). New York: John Wiley.

Suggested Readings

1. Beiser, A. (1987). *Concepts of modern physics,* (4th ed.). New York: McGraw-Hill Book Co.
2. Young, H. D., & Freedman, R. A. (2008). *University physics with modern physics* (14th ed.). London: Pearson.

URCE-5101 **Grammar** **3(3+0)**

The course introduces the students to the underlying rules to acquire & use language in academic context. The course aims at developing grammatical competence of the learners to use grammatical structures in context in order to make the experience of learning English more meaningful enabling the students to meet their real-life communication needs. The objectives of the course are to, reinforce the basics of grammar, underst& the basic meaningful units of language, & introduce the functional aspects of grammatical categories & to comprehend language use by practically working on the grammatical aspects of language in academic settings. After studying the course, students would be able to use the language efficiently in academic & real-life situations & integrate the basic language skills in speaking & writing. The students would be able to work in a competitive environment at higher education level to cater with the long-term learners' needs.

Contents

1. Parts of speech
2. Noun & its types
3. Pronoun & its types
4. Adjective & its types
5. Verb & its types
6. Adverb & its types
7. Prepositions & its types
8. Conjunction & its types
9. Phrases & its different types
10. Clauses & its different types
11. Sentence, parts of sentence & types of sentence
12. Synthesis of sentence
13. Conditional sentences
14. Voices
15. Narration
16. Punctuation
17. Common grammatical errors & their corrections

Recommended Texts

1. Eastwood, J. (2011). *A basic English grammar*. Oxford: Oxford University Press.
2. Swan, M. (2018). *Practical English usage* (8th ed.). Oxford: Oxford University Press.

Suggested Readings

1. Thomson, A. J., & Martinet, A. V. (1986). *A practical English grammar*. Oxford: Oxford University Press
2. Biber, D., Johansson, S., Leech, G., Conrad, S., Finegan, E., & Quirk, R. (1999). *Longman grammar of spoken & written English*. Harlow Essex: MIT Press.
3. Hunston, S., & Francis, G. (2000). *Pattern grammar: A corpus-driven approach to the lexical grammar of English*. Amsterdam: John Benjamins.

URCP-5106 **Pakistan Studies** **2(2+0)**

The course is designed to acquaint the students of BS Programs with the rationale of the creation of Pakistan. The students would be apprised of the emergence, growth & development of Muslim nationalism in South Asia & the struggle for freedom, which eventually led to the establishment of Pakistan. While highlighting the main objectives of national life, the course explains further the socio-economic, political & cultural aspects of Pakistan's endeavours to develop & progress in the contemporary world. For this purpose, the foreign policy objectives & Pakistan's foreign relations with neighbouring & other countries are also included. This curriculum has been developed to help students analyse the socio-political problems of Pakistan while highlighting various phases of its history before & after the partition & to develop a vision in them to become knowledgeable citizens of their home. This course aims to introduce students to the history of the region comprising Pakistan, provide an overview of contending perspectives on the origins of the country, and examine its politics, society and culture. The course, furthermore, looks at some contemporary developmental issues facing the country.

Contents

1. Contextualizing Pakistan Studies
2. Geography of Pakistan: Geo-Strategic Importance of Pakistan
3. Freedom Movement (1857-1947)
4. Pakistan Movement (1940-47)
5. Muslim Nationalism in South Asia
6. Two Nations Theory
7. Ideology of Pakistan
8. Initial Problems of Pakistan
9. Political & Constitutional Developments in Pakistan
10. Economy of Pakistan: Problems & Prospects
11. Society & Culture of Pakistan
12. Foreign Policy Objectives of Pakistan & Diplomatic Relations
13. Current & Contemporary Issues of Pakistan
14. Human Rights: Issues of Human Rights in Pakistan

Recommended Texts

1. Kazimi, M. R. (2007). *Pakistan studies*. Karachi: Oxford University Press.
2. Sheikh, J. A. (2004). *Pakistan's political economic & diplomatic dynamics*. Lahore: Kitabistan Paper Products.

Suggested Readings

1. Hayat, S. (2016). *Aspects of Pakistan movement*. Islamabad: National Institute of Historical & Cultural Research.
2. Kazimi, M. R (2009). *A concise history of Pakistan*. Karachi: Oxford University Press.
3. Talbot, I. (1998). *Pakistan: A modern history*. London: Hurst & Company.

URCI-5109 Introduction to Information and Communication Technologies 3 (2+1)

The course introduces students to information & communication technologies & their current applications in their respective areas. Objectives include basic underst&ing of computer software, hardware, & associated technologies. They can make use of technology to get maximum benefit related to their study domain. Students can learn how the Information & Communications systems can improve their work ability & productivity. How Internet technologies, E-Commerce applications & Mobile Computing can influence the businesses & workplace. At the end of semester, students will get basic underst&ing of Computer Systems, Storage Devices, Operating systems, E-commerce, Data Networks, Databases, & associated technologies. They will also learn Microsoft Office tools that includes Word, Power Point, Excel. They will also learn Open office being used on other operating systems & platforms. Specific software's related to specialization areas are also part of course. The Course will also cover Computer Ethics & related Social media norms & cyber laws.

Contents

1. Introduction, Overview & its types.
2. Hardware: Computer Systems & Components, Storage Devices & Cloud Computing.
3. Software: Operating Systems, Programming & Application Software,
4. Introduction to Programming Language
5. Databases & Information Systems Networks
6. The Hierarchy of Data & Maintaining Data,
7. File Processing Versus Database Management Systems
8. Data Communication & Networks.
9. Physical Transmission Media & Wireless Transmission Media
10. Applications of smart phone & usage
11. The Internet, Browsers & Search Engines.
12. Websites Concepts, Mobile Computing & their applications.
13. Collaborative Computing & Social Networking
14. E-Commerce & Applications.
15. IT Security & other issues

Recommended Texts

1. Vermaat, M. E. (2018). *Discovering computers: digital technology, data & devices.* Boston: Course Technology Press.

Suggested Readings

1. O'Leary, T., O'Leary, L., & O'Leary, D. (2017). *Computing essentials*, (26^{th} ed.). San Francisco: McGraw Hill Higher Education.
2. Schneider, G. M., & Gersting, J. (2018). *Invitation to computer science*. Boston: Cengage Learning.

MATH-5103 **Calculus-II** **3(3+0)**

This is the second course of the basic sequence Calculus serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts & skills needed for mathematical manipulation. As continuation of Calculus-I, it focuses on the study of functions of a single variable. This Core Curriculum course is designed to meet the following four learning goals: Students will construct and evaluate logical arguments. Students will apply and adapt a variety of appropriate strategies to solve mathematical problems. Students will recognize and apply mathematics in contexts outside of mathematics. Students will organize and consolidate mathematical thinking through written and oral communication. Students will integrate transcendental functions, including logarithms, exponential, trigonometry and inverse trigonometric, hyperbolic and inverse hyperbolic functions, apply methods of integration, such as algebraic substitution, trigonometric substitution, partial fractions, integration by parts, and use a table of integrals, solve limit problems involving indeterminate forms with La'Hopital's Rule and convert parametric representation of curves to rectangular coordinates, represent a curve using polar coordinates, and integrate functions expressed in polar coordinates.

Contents

1. Techniques of integration: Using Basic Integration Formulas, Integration by Parts
2. Trigonometric Integrals, Trigonometric Substitutions
3. Integration of Rational Functions by Partial Fractions
4. Integral Tables & Computer Algebra Systems, Numerical Integration, Improper Integrals
5. Sequences & Infinite Series, The Integral Test, Comparison Tests
6. Absolute Convergence, The Ratio & Root Tests
7. Alternating Series & Conditional Convergence
8. Power Series, Taylor & Maclaurin Series, Convergence of Taylor Series
9. The Binomial Series & Applications of Taylor Series
10. Parametrizations of Plane Curves
11. Calculus with Parametric Curves, Polar Coordinates
12. Graphing Polar Coordinate Equations
13. Areas & Lengths in Polar Coordinates, Conic Sections, Conics in Polar Coordinates

Recommended Texts

1. Thomas, G. B., Weir, M. D., & Hass, J. R. (2014). *Thomas' calculus: single variable* (13th ed.). London: Pearson.
2. Stewart, J. (2012). *Calculus*, (8th ed.). New York: Cengage Learning.

Suggested Readings

1. Anton, H., Bivens, I. C., & Davis, S. (2016). *Calculus*, (11th ed.). New York: Wiley.
2. Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed.). London: Pearson.
3. Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed.). New York: Brooks Cole.

MATH-5104 **Statistics** **3(3+0)**

Mathematics & statistics open doors in engineering, business, finance, computing, data sciences, health sciences, environmental sciences & public policy. They are also fascinating in their own right. Recent discoveries in the mathematical sciences have played an essential role in internet search algorithms, disease control, communications technology, climate modelling & much more. Mathematics & Statistics are among the most important disciplines in today's complex world, in part because they serve as the common language of science. The main aim of this course is the study of the statistical distributions like beta, gamma, binomial, exponential, Poisson, hypergeometric & normal distributions. Furthermore, the decision theory & sampling theory is also discussed. Topics discussed in it include displaying and describing data, the normal curve, regression, probability, statistical inference, confidence intervals, and hypothesis tests with applications in the real world. Students also have the opportunity to analyze data sets using technology in their weekly laboratory discussions.

Contents

1. Mathematical Expectation: Moments
2. Moment generating functions
3. Cummulants, cumulative functions
4. Continuous Probability distributions
5. Beta, gamma & binomial distributions
6. Exponential, Poisson, hypergeometric & normal distributions,
7. Sampling distributions
8. Sampling procedures
9. Estimation of parameters
10. Estimation of mean
11. Variance
12. Confidence intervals
13. Hypothesis testing & decision making
14. Types of errors in tests, quality control
15. Control charts for mean
16. Standard deviation**,** Variance, range
17. Goodness of fit, Chi-square test

Recommended Texts

1. Chaudhry, S. M., & Kamal, S. (2008). *Introduction to statistical theory, Part I, II,* (8th ed.). Lahore: Ilmi Kitab Khana.
2. DeGroot, M. H., & Schervish, M. J. (2002). *Probability & statistics* (3rd ed.). Boston: Addison-Wesley.

Suggested Readings

1. Johnson, R. (1994). *Probability & statistics for Engineers* (1st ed.). New Jersey: Prentice-Hall.
2. Papoulis, A. (1991). *Probability, random variables, & stochastic processes*, (3rd ed.). New York: McGraw Hill.
3. Sincich, T. (1990). *Statistics by examples* (1st ed.). San Francisco: Dellen Publication Company.

PHYS- 5162 **Physics-II** **4(3+1)**

This course is continuation of the course Physics-I. This course presents the basic concepts of electricity & magnetism, & certain aspects of electronics. The course objectives are to enable students about the ideas of electric field, electric potential, capacitor, resistance, magnetic field, & some concepts of basic electronics. The course objectives are to enable students about the ideas of electric field, electric potential, capacitor, resistance, magnetic field, & some concepts of basic electronics. Students are encouraged to share their thinking with teachers & peers & to examine different problem-solving strategies, in the said field. At the end of the course, students will be able to conduct qualitative analysis which demonstrates physical and mathematical intuition and conceptual understanding, to perform quantitative calculations in situations involving electric and magnetic fields, and demonstrate knowledge of the relevant basic units, vector addition, and application of basic calculus, to use simple laboratory demonstrations to explain the basic properties of electric and magnetic fields, and electrical circuits.

Contents

1. Electric Field, Gauss's Law, Electric Potential, Current & Resistances
2. Direct Current & Circuits, Capacitors & Dielectrics
3. Inductance, Alternating Current & Circuits Basic Electronics
4. Magnetic Field Effects, Magnetic Properties of Matter
5. Electro-Magnetic Waves (Maxwell's Equations)
6. *Lab-II*
7. Conversion of a galvanometer into Voltmeter & an Ammeter,
8. To determine the frequency of A.C mains by using a sonometer,
9. To determine the frequency of A.C by Meld's experiment,
10. Resonance frequency of an acceptor circuit, Resonance frequency of a rejector circuit,
11. To set up & study various logic gates (OR,&, NOT, N& etc.) using diode & to develop their truth table,
12. Study the characteristics of a transistor.

Recommended Texts:

1. Raymond, A., & Jewett, J. W. (2011). *Physics for scientists & engineers with modern physics* (8th ed.). New York: Cengage Learning.
2. Halliday, D., Resnick, R., & Walker, J. (2008). *Fundamental of physics, extended* (8th ed.). New York: John Wiley.

Suggested Readings

1. Reitz, John, R., & Fredrick, M. J. (1970). *Foundations to electromagnetic theory* (2nd ed.). Boston: Addison-Wesley Publishing Co.
2. Young, H. D., & Freedman, R. A. (2008). *University physics with modern physics* (14th ed.). London: Pearson.
3. Grobe. (1993). *Basic electronics* (7th ed.). New York: McGraw Hill Book Co.

URCE-5102 **Language Comprehension & Presentation Skills** **3(3+0)**

The course aims at developing linguistic competence by focusing on basic language skills in integration to make the use of language in context. It also aims at developing students' skills in reading & reading comprehension of written texts in various contexts. The course also provides assistance in developing students' vocabulary building skills as well as their critical thinking skills. The contents of the course are designed based on these language skills: listening skills, pronunciation skills, comprehension skills & presentation skills. The course provides practice in accurate pronunciation, stress & intonation patterns & critical listening skills for different contexts. The students require a grasp of English language to comprehend texts as organic whole, to interact with reasonable ease in structured situations, & to comprehend & construct academic discourse. The course objectives are to enhance students' language skill management capacity, to comprehend text(s) in context, to respond to language in context, & to write structured response(s).

Contents

1. Listening skills
2. Listening to isolated sentences & speech extracts
3. Managing listening & overcoming barriers to listening
4. Expressing opinions (debating current events) & oral synthesis of thoughts & ideas
5. Pronunciation skills
6. Recognizing phonemes, phonemic symbols & syllables, pronouncing words correctly
7. Underst&ing & practicing stress patterns & intonation patterns in simple sentences
8. Comprehension skills
9. Reading strategies, summarizing, sequencing, inferencing, comparing & contrasting
10. Drawing conclusions, self-questioning, problem-solving, relating background knowledge
11. Distinguishing between fact & opinion, finding the main idea, & supporting details
12. Text organizational patterns, investigating implied ideas, purpose & tone of the text
13. Critical reading, SQ3R method
14. Presentation skills, features of good presentations, different types of presentations
15. Different patterns of introducing a presentation, organizing arguments in a presentation
16. Tactics of maintaining interest of the audience, dealing with the questions of audience
17. Concluding a presentation, giving suggestions & recommendations

Recommended Texts

1. Mikulecky, B. S., & Jeffries, L. (2007). *Advanced reading power: Extensive reading, vocabulary building, comprehension skills, reading faster*. New York: Pearson.
2. Helgesen, M., & Brown, S. (2004). *Active listening: Building skills for underst &ing*. Cambridge: Cambridge University Press.

Suggested Readings

1. Roach, C. A., & Wyatt, N. (1988). *Successful listening*. New York: Harper & Row.
2. Horowitz, R., & Samuels, S. J. (1987). *Comprehending oral & written language*. San Diego: Academic Press.

URCI-5105 **Islamic Studies** **2(2+0)**

Islamic Studies engages in the study of Islam as a textual tradition inscribed in the fundamental sources of Islam, Qur'an & Hadith, history & particular cultural contexts. The area seeks to provide an introduction to & a specialization in Islam through a large variety of expressions (literary, poetic, social, & political) & through a variety of methods (literary criticism, hermeneutics, history, sociology, & anthropology). It offers opportunities to get fully introductory foundational bases of Islam in fields that include Qur'anic studies, Hadith & Seerah of Prophet Muhammad (PBUH), Islamic philosophy, & Islamic law, culture & theology through the textual study of Qur'an & Sunnah. Islamic Studies is the academic study of Islam & Islamic culture. It majorly comprises of the importance of life & that after death. It is one of the best systems of education, which makes an ethical groomed person with the qualities which he/she should have as a human being. The basic sources of the Islamic Studies are the Holy Qur'an & Sunnah or Hadith of the Holy Prophet Muhammadﷺ. The learning of the Qur'an & Sunnah guides the Muslims to live peacefully.

Contents

1. Study of the Qur'an (Introduction to the Qur'an, Selected verses from *Surah Al-Baqarah, Al-Furqan, Al-Ahzab, Al-Mu'minoon, Al-An'am, Al-Hujurat, Al-Saff*)
2. Study of the Hadith (Introduction to Hadith literature, Selected Ahadith (Text & Translation)
3. Introduction to Qur'anic Studies
4. Basic Concepts of Qur'an
5. History of Quran, Basic Concepts of Hadith
6. History of Hadith
7. Kinds of Hadith
8. Uloom –ul-Hadith
9. Sunnah & Hadith
10. Seerat ul-Nabi (PBUH), necessity & importance of Seerat, role of Seerah in the development of personality, Pact of Madinah, Khutbah Hajjat al-Wada' & ethical teachings of Prophet (PBUH).
11. Legal Position of Sunnah
12. Islamic Culture & Civilization
13. Characteristics of Islamic Culture & Civilization
14. Historical Development of Islamic Culture & Civilization
15. Comparative Religions & Contemporary Issues
16. Impact of Islamic civilization

Recommended Books

1. Hassan, A. (1990). *Principles of Islamic jurisprudence*. New Dehli: Adam Publishers.
2. Zia-ul-Haq, M. (2001). *Introduction to al-Sharia al-Islamia*. Lahore: Aziz Publication.

Suggested Readings

1. Hameedullah, M. (1957). *Introduction to Islam*. Lahore: Sh M Ashraf Publisher.
2. Hameedullah, M. (1980). *Emergence of Islam*. New Dehli: Adam Publishers.
3. Hameedullah, M. (1942). *Muslim conduct of state.* Lahore: Sh M Ashraf Publisher.

MATH-5105 Programming Languages for Mathematicians 3(2+1)

Programming Languages plays an important role in Mathematics. More often, the act of programming involves problem-solving in itself, where you then take your answers and apply them to build a program. However, mathematicians sometimes require some programming languages for assistance, and some of the best programming languages for math work wonders when you're trying to hone your skills and train yourself in a particular mathematical field. A number of computer software available to deal with mathematical computing & simulation. This course provides a practical introduction to most widely used Mathematical computing software's namely, MATHEMATICA or MAPLE. Maple has a fairly strong advantage when it comes to combinatorial math problems. It's also known for its functional programming constructs, making it extremely interesting to play around with. After this course students will be able to develop computer programs in this software according to their requirements in mathematical computing. It includes ntroduction to data-oriented Python packages, decision trees, support vector machines (SVM), neural networks, and machine learning.

Contents

Mathematica
1. Introduction to the basic environment of MATHMATICA & its syntax
2. Running MATHEMATICA
3. Numerical/Algebraic Calculations, vectors, Matrices, Sets, Lists, Tables, arrays
4. Symbolic Mathematics in MATHEMATICA
5. Functions & functional programming
6. Procedural programming, Do, for & while loops, Flow controls
7. Graphics, Plots of 2D & 3D functions, Packages within MATHEMATICA

Maple
1. Introductory Demonstration of Maple, symbolic computations in MAPLE
2. Vectors, Matrices, Sets, Lists, Tables, arrays & Arrays, Toolbars & Palettes
3. Operators, Constant, Elementary Functions, Procedures
4. If clauses, selection & conditional execution
5. Looping, for & while loop, looping commands, recursion
6. Plots of 2D & 3D functions, Packages within MAPLE

Recommended Texts

1. Wellin, P., Kamin, S., & Gaylord R. (2011). *An introduction to programming with mathematica*, (3rd ed.). Cambridge: Cambridge University Press.
2. Monagan, M. B., & Geddes, K. O. (2005). *Maple introductory programming guide*. Waterloo: Maplesoft, a division of Waterloo Maple Inc.

Suggested Readings

1. Aladjev, V. Z., & Bogdivicus, M. A. (2006). *Maple: Programming, physical & engineering Problems*. London: Fultus Publishing.
2. Maeder, R. E. (1997). *Programming in mathematica* (3rd ed.). Boston: Addision-Weseley.
3. Hoste, J. (2009). *Mathematica demystified*. New York: McGraw Hill.

URCC-5110 Citizenship Education and Community Engagement 3 (1+2)

In recent years, community engagement has become a central dimension of governance as well as policy development & service delivery. However, efforts to directly involve citizens in policy processes have been bedeviled by crude underst&ings of the issues involved, & by poor selection of techniques for engaging citizens. This course will provide a critical interrogation of the central conceptual issues as well as an examination of how to design a program of effective community engagement. This course begins by asking: Why involve citizens in planning & policymaking? This leads to an examination of the politics of planning, conceptualizations of "community" &, to the tension between local & professional knowledge in policy making. This course will also analyze different types of citizen engagement & examine how to design a program of public participation for policy making. Approaches to evaluating community engagement programs will also be a component of the course. Moreover, in order to secure the future of a society, citizens must train younger generations in civic engagement & participation. Citizenship education is education that provides the background knowledge necessary to create an ongoing stream of new citizens participating & engaging with the creation of a civilized society.

Contents

1. Introduction to Citizenship Education & Community Engagement: Orientation
2. Introduction to Active Citizenship: Overview of the ideas, Concepts, Philosophy & Skills
3. Identity, Culture & Social Harmony: Concepts & Development of Identity
4. Components of Culture & Social Harmony, Cultural & Religious Diversity
5. Multi-cultural society & inter-cultural dialogue: bridging the differences, promoting harmony
6. Significance of diversity & its impact, Importance & domains of inter-cultural harmony
7. Active Citizen: Locally active, Globally connected
8. Importance of active citizenship at national & global level, understading community
9. Identification of resources (human, natural & others), Human rights, Universalism vs relativism
10. Constitutionalism & citizens' responsibilities: Introduction to human rights
11. Human rights in constitution of Pakistan, Public duties & responsibilities
12. Social Issues in Pakistan: Introduction to the concept of social problem, Causes & solutions
13. Social Issues in Pakistan (Poverty, Equal & Equitable access of resources, unemployment)
14. Social Issues in Pakistan (Agricultural problems, terrorism & militancy, governance issues)
15. Social action & project: Introduction & planning of social action project
16. Identification of problem, Ethical considerations related to project, Assessment of existing resources

Recommended Books

1. Kennedy, J. K., & Brunold, A. (2016). *Regional context & citizenship education in Asia & Europe*. New York: Routledge Falmer.
2. Macionis, J. J., & Gerber, M. L. (2010). *Sociology*. New York: Pearson Education

Suggested Books

1. Council, B. (2017). *Active citizen's social action projects guide.* Scotland: British Council.
2. Larsen, K. A. (2013). *Participation in community work: international perspectives.* Vishanthic Sewpaul, Grete Oline Hole.

MATH-5106 **Calculus-III** **3(3+0)**

This is the third course of the basic sequence Calculus-1, II & III, serving as the foundation of advanced subjects in all areas of mathematics. Its focus on the study of functions of a multivariable. The main focus of the course is to the study of multiple integrals in different coordinate systems & their applications. Moreover, a brief introduction to vector calculus will also be presented. It plays a role in our understanding of fundamental particles, the structure of crystal lattices and the geometry of molecules. In this course, we will begin by defining the axioms satisfied by groups and begin to develop basic group theory by reference to some elementary examples. We will analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons and regular solids, and groups of matrices. We will develop the notions of homomorphism, normal subgroups and quotient groups and study the First Isomorphism Theorem and its application.

Contents

1. Vectors & analytic geometry in space: Three-dimensional Coordinate System
2. Vectors, lines & planes in space
3. The dot product, the cross product
4. Cylinder & Quadric surfaces, vector-valued functions
5. Vector functions & space curve
6. Derivatives & integrals of vector functions
7. Arc length & Curvature
8. Motion in space, Velocity & Acceleration
9. Tangential & Normal Components of Acceleration
10. Velocity & Acceleration in Polar Coordinates
11. Functions of several variables, limits, Continuity & partial derivatives
12. Chain rule, directional derivatives & the gradient vector
13. Maximum & minimum values, optimization problems, Lagrange Multipliers
14. Multiple integrals: Double integrals over rectangles & iterated integrals
15. Double integrals over general regions
16. Double integrals in polar coordinates
17. Triple integrals in rectangular, cylindrical & spherical coordinates

Recommended Texts

1. Thomas, G. B., Weir, M. D., & Hass J. R. (2014). *Thomas' Calculus: multivariable* (13th ed.). London: Pearson.
2. Stewart, J. (2015). *Calculus* (8th ed.). New York: Cengage Learning.

Suggested Readings

1. Anton, H., Bivens, I. C., & Davis, S. (2016). *Calculus* (11th ed.). New York: Wiley.
2. Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed.). London: Pearson.
3. Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed.). New York: Brooks Cole.

MATH-5107 **Algebra-I** **3(3+0)**

This course is is an introduction to group theory, one of the three main branches of pure mathematics. Group theory is the study of groups. Group theory is one of the great simplifying and unifying ideas in modern mathematics. It was introduced in order to understand the solutions to polynomial equations, but only in the last one hundred years has its full significance, as a mathematical formulation of symmetry, been understood. It plays a role in our understanding of fundamental particles, the structure of crystal lattices and the geometry of molecules. In this course, we will begin by defining the axioms satisfied by groups and begin to develop basic group theory by reference to some elementary examples. We will analyse the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons and regular solids, and groups of matrices. We will develop the notions of homomorphism, normal subgroups and quotient groups and study the First Isomorphism Theorem and its application.

Contents

1. Groups, definition & examples of groups, elementary properties of groups
2. Finite & Infinite Groups
3. Order of element of a group & related results
4. Subgroups, examples of subgroup, subgroup tests, subgroup generated by set
5. Cyclic groups, properties of cyclic groups
6. Classification of subgroups of cyclic groups
7. Cosets decomposition of a group, properties of cosets
8. Lagrange's theorem & its consequences
9. Conjugate elements & conjugacy classes
10. Centralizer of a subset of a group, normalizer of a subset of a group
11. Center of group definition & examples
12. Normal Subgroups, factor groups, application of factor groups
13. Permutations & Permutation groups, definition & examples
14. Homomorphism of groups, properties of Homomorphisms

Recommended Texts

1. Gallian, J. A. (2017). *Contemporary abstract algebra* (9th ed.). New York: Brooks/Cole.
2. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.

Suggested Readings

1. Roman, S. (2012). *Fundamentals of group theory* (1st ed.). Basel: Birkhäuser.
2. Rose, H. E. (2006). *A course on finite groups* (1st ed.). London: Springer-Verlag.
3. Fraleigh, J. B. (2003). A *first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.

PHYS-5163 **Physics-III** **4(3+1)**

This is the third course of the basic sequence Physics-I, II & III. This course aims to introduce students with Newtonian's Mechanics as well as Waves & Oscillations. This course wills also describe the basic concepts of Vectors, Power & Energy. This course will also cover the three phenomena of diffraction, interference & polarization. In addition, in the laboratory they will also familiar with Modulus of rigidity by static & dynamic method (Maxwell's needle, Bartons Apparatus), Determination of moment of inertia of a solid/hollow cylinder & a sphere etc. & with study the conservtion of energy (Hook's Law). It plays a role in our understanding of fundamental particles, the structure of crystal lattices and the geometry of molecules. In this course, we will begin by defining the axioms satisfied by groups and begin to develop basic group theory by reference to some elementary examples.

Contents

1. Kinetic theory of gases, The Van der Waals equation
2. Temperature, Thermodynamics & Thermodynamic equilibrium
3. Maxwell distribution of molecular speeds & Energies
4. Laws of Thermodynamics
5. Heat engine, Carnot cycle & efficiency measurements, Concept of entropy
6. Entropy measurements for reversible & irreversible process
7. Low temperature Physics
8. Thermodynamic relations
9. Thermoelectricity
10. Basic principles of Statistical Mechanics
11. Microscopic & macroscopic states, Phase space
12. Partition function, Relations of partition
13. Function with thermo dynamical variables
14. *Lab-III*
15. Measurement of resistance using a Neon flash bulb & condenser
16. Determination of ionization potential of mercury
17. To determine the stopping potential by photocell

Recommended Texts:

1. Garg, S. C., Bansal, R. M., & Ghosh, C. K. (2013). *Thermal physics kinetic theory, thermodynamics & statistical mechanics* (2nd ed.). New York: McGraw-Hill Education Private Limited.
2. Raymond, A., & Jewett, J. W. (2011). *Physics for scientists & engineers with modern physics* (8th ed.). New York: Cengage Learning.

Suggested Readings

1. Young, H. D., & Freedman, R. A. (2008). *University physics with modern physics* (14th ed.). London: Pearson.
2. Beiser, A. (1987). *Concepts of modern physics* (4th ed.). New York: McGraw-Hill Book Co.

URCE-5103 **Academic Writing** **3 (3+0)**

Academic writing is a formal, structured & sophisticated writing to fulfill the requirements for a particular field of study. The course aims at providing underst&ing of writer's goal of writing (i.e. clear, organized & effective content) & to use that underst&ing & awareness for academic reading & writing. The objectives of the course are to make the students acquire & master the academic writing skills. The course would enable the students to develop argumentative writing techniques. The students would be able to the content logically to add specific details on the topics such as facts, examples & statistical or numerical values. The course will also provide insight to convey the knowledge & ideas in objective & persuasive manner. Furthermore, the course will also enhance the students' underst&ing of ethical considerations in writing academic assignments & topics including citation, plagiarism, formatting & referencing the sources as well as the technical aspects involved in referencing.

Contents

1. Academic vocabulary
2. Quoting, summarizing & paraphrasing texts
3. Process of academic writing
4. Developing argument
5. Rhetoric: persuasion & identification
6. Elements of rhetoric: Text, author, audience, purposes, setting
7. Sentence structure: Accuracy, variation, appropriateness, & conciseness
8. Appropriate use of active & passive voice
9. Paragraph & essay writing
10. Organization & structure of paragraph & essay
11. Logical reasoning
12. Transitional devices (word, phrase & expressions)
13. Development of ideas in writing
14. Styles of documentation (MLA & APA)
15. In-text citations
16. Plagiarism & strategies for avoiding it

Recommended Texts

1. Swales, J. M., & Feak, C. B. (2012). *Academic writing for graduate students: Essential tasks & skills* (3rd ed.). Ann Arbor: The University of Michigan Press.
2. Bailey, S. (2011). *Academic writing: A h&book for international students* (3rd ed.). New York: Routledge.

Suggested Readings

1. Craswell, G. (2004). *Writing for academic success*. London: SAGE.
2. Johnson-Sheehan, R. (2019). *Writing today*. Don Mills: Pearson.
3. Silvia, P. J. (2019). *How to write a lot: A practical guide to productive academic writing*. Washington: American Psychological Association.

MATH-5108 **Probability Theory** **3(3+0)**

This course is the introduction to the Probability theory, which is the branch of mathematics that deals with modelling uncertainty. It is important because of its direct application in areas such as genetics, finance and telecommunications. It also forms the fundamental basis for many other areas in the mathematical sciences including statistics, modern optimisation methods and risk modelling. A prime objective of the course is to introduce the students to the fundamentals of probability theory & present techniques & basic results of the theory & illustrate these concepts with applications. This course will also present the basic principles of random variables & random processes needed in 24 applications. After the course, the students should be able to understand axioms of Probability theory and have working knowledge with probability calculations behind hierarchical model building; be aware of different types of convergence and to be able to visualize basic concepts of probability theory using Matlab.

Contents

1. Finite probability spaces
2. Basic concept
3. probability & related frequency
4. Combination of events
5. Some examples of Combination of events
6. Independence, random variables, expected value, standard deviation
7. Chebyshev's inequality
8. Independence of r&om variables
9. Multiplicatively of the expected value
10. Additivity of the variance
11. Discrete probability distribution
12. Probability as a continuous set function
13. Sigma-algebras, examples
14. Continuous random variables, expectation & variance
15. Normal random variables & continuous probability distribution
16. Applications: De Moivre-Laplace limit theorem
17. Weak & strong law of large numbers
18. The central limit theorem
19. Markov chains & continuous Markov process

Recommended Texts

1. Capinski, M., & Kopp, E. (1998). *Measure, integral & probability.* London: Springer-Verlag.
2. Dudley, R. M. (2004). *Real analysis & probability.* Cambridge: Cambridge University Press.

Suggested Readings

1. Resnick, S. I. (1999). *A probability path*, Basel: Birkhauser.
2. Ross, S. (1998). *A first course in probability theory* (5th ed.). New Jersey: Prentice Hall.
3. Ash, R. B. (2008). *Basic probability theory*. New York: Dover Books.

MATH-5109 Vector Analysis & Mechanics 3(3+0)

This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics. This course introduces the fundamental principles in mechanics. Structural design applications of a variety of problems are developed throughout the course using examples that elucidate the theory of mechanics. It emphasizes on the laws of friction, equilibrium, center of gravity & harmonic & orbital motion. The objectives of the course are to develop better understanding of key concepts concerning scalar and vector fields learned previously in Multivariable Calculus courses, to gain deeper knowledge of multivariate differentiation operations such as Gradient, Divergent and Curl, master the Integral Theorems at the core of Vector Analysis: the Stokes (Greens') Theorem and the Divergence (Gauss') Theorem and to learn the utility of Vector Analysis by learning its relevance to Maxwell's equations describing the dynamics of electric and magnetic fields. In this course, students are prepared for further study in the relevant technological disciplines and more advanced mathematics courses.

Contents

1. Vector Analysis: Scalar & vector triple product
2. Differentiation & integration of vector functions
3. Gradient, divergence, Curl of vector point functions
4. Mechanics: Composition & resolution of co-planar forces, Moments
5. Couples & conditions of equilibrium under the action of co-planar forces
6. Frictional forces, Laws of friction
7. Equilibrium of bodies on rough surfaces
8. Principle of virtual work & related problems
9. Center of gravity, Center of mass of various bodies
10. Kinematics of a particle in Cartesian & polar co-ordinates
11. Linear & angular velocity
12. Rectilinear motion with uniform & variable acceleration
13. Simple harmonic motion, Projectile motion
14. Motion along horizontal & vertical circles
15. Orbital motion, planetary motion & keplar' laws, conservative forces
16. Damped forces

Recommended Texts

1. Munawar, H., Saeed, S. M., & Ahmed, C. B. (2016). *Elementary vector analysis*. Lahore: The Caravan Book House.
2. Ghori, Q. K. (2015). *Mechanics*. Lahore: West Pakistan Publishing Company.

Suggested Readings

1. Spiegel, M. R., Lipschutz, S., & Spellman, D. (2009). *Schaum's outline vector analysis* (2nd ed.). New York: McGraw-Hill Education.
2. Brand, L. (2006). *Vector analysis*. New York: Dover Publications.
3. Yousuf, S. M. (1988). *Vector analysis*. Lahore: Ilmi Ketab Khana.

MATH-5110 **Linear Algebra** **3(3+0)**

Linear algebra is the study of linear systems of equations, vector spaces, and linear transformations. Solving systems of linear equations is a basic tool of many mathematical procedures used for solving problems in science and engineering. Linear Algebra plays a significant role in many areas of mathematics, statistics, engineering, the natural sciences, and the computer sciences. It provides a foundation of important mathematical ideas that will help students be successful in future coursework. The main objective of this course is to help students to learn in rigorous manner, the tools & methods essential for studying the solution spaces of problems in mathematics and in other fields & develop mathematical skills needed to apply these to the problems arising within their field of study and to various real-world problems. The student will become competent in solving linear equations, performing matrix algebra, calculating determinants, finding eigenvalues & eigenvectors and the student will come to understand a matrix as a linear transformation relative to a basis of a vector space.

Contents

1. Representation of linear equations in matrix form
2. Solution of linear system, Gauss-Jordan & Gaussian elimination method
3. Vector space, definition, examples & properties
4. Subspaces, Linear combination & spanning set
5. Linearly Dependent & Linearly Independent sets
6. Bases & dimension of a vector space
7. Intersections, sums & direct sums of subspaces, Quotient Spaces, Change of basis
8. Linear transformation, Rank & Nullity of linear transformation
9. Matrix of linear transformations
10. Eigen values & eigen vectors, Dual spaces
11. Inner product Spaces with properties, Projection
12. Cauchy inequality
13. Orthogonal & orthonormal basis
14. Gram Schmidt process & diagonalization

Recommended Texts

1. Dar, K. H. (2007). *Linear algebra* (1st ed.). Karachi: The Carwan Book House.
2. Kolman, B., & Hill, D. R. (2005). *Introductory linear algebra* (8th ed.). London: Pearson/Prentice Hall.

Suggested Readings

1. Cherney, D., Denton, T., Thomas, R., & Waldron, A. (2013). *Linear algebra* (1st ed.). California: Davis.
2. Anton, H., & Rorres, C. (2014). *Elementary linear algebra: applications version* (11th ed.). New York: John Wiley & Sons.
3. Grossman, S. I. (2004). *Elementary linear algebra* (5th ed.). New York: Cengage Learning.

PHYS- 5164 **Physics-IV** **4(3+1)**

This course presents the basic concepts of relativistic mechanics & Quantum theory along with its mechanics. The course objectives are to enable students about the ideas of atomic Physics, structure of nuclear & brief introduction to Cosmology. Students are encouraged to share their thinking with teachers & peers & to examine different problem-solving strategies, in the said field. The main objective of this course is to help students to learn in rigorous manner, the tools & methods essential for studying the solution spaces of problems in mathematics and in other fields & develop mathematical skills needed to apply these to the problems arising within their field of study and to various real-world problems. The student will become competent in solving linear equations, performing matrix algebra, calculating determinants, finding eigenvalues & eigenvectors and the student will come to understand a matrix as a linear transformation relative to a basis of a vector space.

Contents

1. Relativistic mechanics, Origin of Quantum Theory
2. Wave nature of matter, Quantum Mechanics
3. Introduction to Quantum Optics (Laser) & Plasma Physics
4. Atomic Physics, bonding in solids, b& theory of solids
5. Fundamental forces in nature
6. Nuclear structure, fundamental particles
7. Nuclear transmutation (Alpha-Beta & Gamma decays)

Lab-IV

1. Determination of e/m of an electron,
2. Characteristics of a semiconductor diode,
3. Setting up of half & full wave rectifier & study of following factors
4. Smoothing effect of a capacitor, Ripple factor & its variation with load
5. Study of regulation of output voltage with load,
6. Study of the parameter of wave i.e. amplitude, phase & time period of a complex signal by

Recommended Texts

1. Halliday, D., Resnick, R., & Walker, J. (2008). *Fundamental of physics, extended* (8th ed.). New York: John Wiley.
2. Raymond, A., & Jewett, J. W. (2011). *Physics for scientists & engineers with modern physics* (8th ed.). New York: Cengage Learning.

Suggested Readings

1. Reitz, John, R., & Fredrick, M. J. (1970). *Foundations to electromagnetic theory* (2nd ed.). Boston: Addison-Wesley Publishing Co.
2. Young, H. D., & Freedman, R. A. (2008). *University physics with modern physics* (14th ed.). London: Pearson.
3. Krane, K. S. (1987). *Introductory nuclear physics* (3rd ed.). New York: Wiley.

MATH-5111 Discrete Mathematics 3(3+0)

This is an introductory course in discrete mathematics. Discrete Mathematics is study of distinct, un-related topics of mathematics; it embraces topics from early stages of mathematical development & recent additions to the discipline as well. It is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics, such as integers, graphs, & statements in logic. The goal of this course is to introduce students to ideas and techniques from discrete mathematics that are widely used in science and engineering. This course teaches the students techniques in how to think logically and mathematically and apply these techniques in solving problems. To achieve this goal, students will learn logic and proof, sets, functions, as well as algorithms and mathematical reasoning. Key topics involving relations, graphs, trees, and formal languages and computability are covered in this course. The present course restricts only to counting methods, relations & graphs. The objective of the course is to inculcate in the students the skills that are necessary for decision making in non-continuous situations.

Contents

1. Counting methods: Basic methods: product
2. inclusion-exclusion formulae
3. Permutations & combinations
4. Recurrence relations & their solutions
5. Generating functions
6. Double counting & its pplications
7. Pigeonhole principle & its applications
8. Relations: Binary relations, n-ary Relations, closures of relations
9. Composition of relations, inverse relation
10. Graphs: Graph terminology
11. Representation of graphs
12. Graphs isomorphism
13. Algebraic methods: the incidence matrix, connectivity
14. Eulerian & Hamiltonian paths, shortest path problem
15. Trees & spanning trees, Complete graphs & bivalent graphs

Recommended Texts

1. Rosen, K. H. (2012). *Discrete mathematics & its applications.* New York: The McGraw-Hill Companies, Inc.
2. Chartr, G., & Zhang, P. (2012). *A first course in graph theory.* New York: Dover Publications, Inc.

Suggested Readings

1. Tucker, A. (2002). *Applied combinatorics.* New York: John Wiley & Sons.
2. Diestel, R. (2010). *Graph theory* (4^{th} ed.). New York: Springer- Verlag
3. Brigs, N. L. (2003). *Discrete mathematics.* Oxford: Oxford University Press.

MATH-5112 **Spanish Language** **3(3+0)**

Spanish is a Romance language that originated in the Iberian Peninsula & today is a global language with more than 483 million native speakers, mainly in Spain & the Americas. It is the world's second-most spoken native language, after M&arin Chinese & the world's fourth-most spoken language, after English, M&arin Chinese & Hindi. Spanish is a part of the Ibero-Romance group of languages, which evolved from several dialects of Vulgar Latin in Iberia after the collapse of the Western Roman Empire in the 5th century. The oldest Latin texts with traces of Spanish come from mid-northern Iberia in the 9th century, & the first systematic written use of the language happened in Toledo, a prominent city of the Kingdom of Castile, in the 13th century. Beginning in 1492, the Spanish language was taken to the viceroyalties of the Spanish Empire, most notably to the Americas, as well as territories in Africa, Oceania & the Philippines. Spanish course develops the ability to communicate directly & effectively with people from Spanish culture. The focus of the curriculum is the progressive development of the skills of listening, speaking, reading & writing in the Spanish language. This course will be fruitful for the students who are seeking opportunities for higher studies in Spanish countries.

Contents

1. Identify Spanish alpha batiks sounds
2. Identify numbers
3. Listening
4. Speaking
5. Reading
6. Writing
7. Underst&ing description about daily life
8. Basic Grammar Rules
9. Total Physical Response
10. Storytelling materials
11. Novels
12. Newspapers (Spanish)
13. Media
14. Classes discussion
15. Magazines

Recommended Texts

1. Vargas, D. C. (2008). *The big red book of Spanish.* New York: McGraw-Hill.
2. Bregstein, B. (2015). *Easy spanish step by step.* Seattle: Amazon Publisher.

Suggested Readings

1. Sanchez, C. (2017). *Spanish short stories for beginners*. (Kindle Ed.). Seattle: Amazon Publisher.
2. Bregstein, B. (2015). *Easy Spanish step by step.* Seattle: Amazon Publisher.

MATH-5113 **French Language** **3(3+0)**

In modern times, French is still a significant diplomatic language: it is an official language of the United Nations, the Olympic Games, & the European Union. It is also the official language of 29 countries. The objective for complete beginners is to reach next level. At this stage the student will be able to underst& very simple phrases about daily life if someone speaks to him slowly & clearly. The student will know how to introduce himself, to describe his work, to talk about his tastes & his hobbies & to describe his past. He will be able to respond to simple & practical questions. The course if designed to teach the basic levels of French Language. It covers the content of the situation which corelates to the corresponding needs of the students. It also addresses the needs of the country, which seeks to master the requirements of the situaiotn.

Contents

1. Actes De Communication
2. Saluer et prendrecongé
3. Dem&er et donnerl'identitéd'unepersonne
4. Faire comprendrequ'onn'a pas compris
5. Se présenter de façoninformelle
6. Épeler, Téléphoner, Aborderquelqu'un, Situer et se situerdansl'espace et dans le temps
7. Fixer des rendez-vous, Donner un emploi du temps, Inviter, accepter et refuserune invitation Offrir et remercier, Décrire un lodgement
   MORPHOSYNTAXE:
8. Les articles définis, indéfinis, contractés et partitifs
9. Conjugaison à l'indicatifprésent des verbes du 1er et du 2èmegroupes
10. Conjugaison à l'indicatifprésent des auxiliaires et de quelquesirréguliers : faire, venir, aller, prendre, partir, sortir, dormir, servir, sentir, suivre, savoir, devoir, vouloir et pouvoir
11. Les pronomspersonnelssujets 1ère et 2èmeformes (atones et toniques)
12. Les formes interrogative et négative, Le genre et le nombre des noms et adjectifs
13. Le passé composé, Le futurproche, Les principalesprépositions et adverbes de lieu
    Phonetique
14. Les phonèmes du français, L'intonationdans les phrases affirmative
15. Négative et interrogative, Les accents et groupesrythmiques
16. Distinction entre les différents sons vocaliques
    Lexique

Recommended Texts

1. Cossard, M., & Salazar, R. (1980). *French basic course*. USA: Foriegn Service Institute.
2. Boulares, M., & Frerot, J. L. (1999). *Grammaire progressive du français*. Cleveland: CLE Internationale.

Suggested Readings

1. Studio 100, niveau 1, méthode de français.
2. Boulares, M., & Frerot, J. L. *Grammaire progressive du français*. Cleveland: CLE Internationale.

ECON-5118 **Mathematical Economics** **3(3+0)**

The course is intended for students who are wishing to obtain knowledge of mathematical techniques suitable for economic analysis. It assumes very little prerequisite knowledge. The approach is informal & aims to show students how to do & apply the mathematics practically. Economic applications are considered although this course aims to teach the mathematics rather than the economics. Topics include basic algebra, simple finance, calculus & matrix algebra. The central concepts of topology, compactness, connectedness & separation axioms are introduced. Applications of topology to number theory, algebraic geometry, algebra & functional analysis are featured. Since many important applications of topology use metric spaces, we investigate topological concepts applied to them & introduce the notion of completeness. In addition, this course provides the basis for studying differential geometry, functional analysis, classical & quantum mechanics, dynamical systems, algebraic & differential topology.

Contents

1. Economic Applications of Graphs & Equations: Isocost Lines, Supply & Dem& Analysis
2. Income Determination Models, IS-LM Analysis
3. Uses of the Derivative in Mathematics & Economics: Increasing & Decreasing Functions
4. Concavity & Convexity
5. Relative Extrema, Inflection Points, Optimization of Functions
6. Successive-Derivative Test for Optimization, Marginal Concepts
7. Optimizing Economic Functions, Relationship among Total
8. Marginal, & Average Concepts
9. Calculus of Multivariable Functions in Economics: Marginal Productivity
10. Income Determination, Multipliers & Comparative Statics
11. Income & Cross Price Elasticities of Dem&, Differentials & Incremental Changes
12. Optimization of Multivariable Functions in Economics
13. Constrained Optimization of Multivariable Functions in Economics
14. Homogeneous Production Functions, Returns to Scale
15. Optimization of Cobb-Douglas Production Functions
16. Optimization of Constant Elasticity of Substitution Production Functions

Recommended Texts

1. Dowling, E.T. (2001). *Introduction to mathematical economists, Schaum's outline series* (3rd ed.). New York: McGraw Hill Publishing Company.
2. Weber, E.J. (1976). *Mathematical analysis, business & economic application*. New York: Harper & Row Publishers.

Suggested Readings

1. Chiang, A.C., & Wainwright, K. (2005). *Fundamental methods of mathematical economics* (4th ed.). New York: McGraw Hill Publishing Company.
2. Frank, B.N. (1993). *Applied mathematics for business, economics & social sciences* (4th ed.). New York: McGraw Hill Publishing Company.

MATH-6112 **Topology** **3(3+0)**

Topology studies continuity in its broadest context. We begin by analyzing the notion of continuity familiar from calculus, showing that it depends on being able to measure distance in Euclidean space. This leads to the more general notion of a metric space. A brief investigation of metric spaces shows that they do not provide the most suitable context for studying continuity. A deeper analysis of continuity in metric spaces shows that only the open sets matter, which leads to the notion of a topological spaces. We easily see that this is the right setting for studying continuity. The central concepts of topology, compactness, connectedness & separation axioms are introduced. Applications of topology to number theory, algebraic geometry, algebra & functional analysis are featured. Since many important applications of topology use metric spaces, we investigate topological concepts applied to them & introduce the notion of completeness. In addition, this course provides the basis for studying differential geometry, functional analysis, classical & quantum mechanics, dynamical systems, algebraic & differential topology.

Contents

1. Topological spaces
2. Bases & sub-bases
3. First & second axiom of countability
4. Separability
5. Continuous functions & homeomorphism
6. Finite product space
7. Separation axioms (T_0)
8. Separation axioms (T_1)
9. Separation axioms (T_2)
10. Techonoff spaces
11. Regular spaces
12. Completely regular spaces
13. Normal spaces
14. Product spaces
15. Compactness
16. Connectedness

Recommended Texts

1. Sheldon, W. D. (2005). *Topology* (1st ed.). New York: McGraw Hill.
2. Willard, S. (2004). *General topology* (1st ed.). New York: Dover Publications.

Suggested Readings

1. Lipschutz, S. (2011). *General topology*, *Schaum's outline series* (1st ed.). New York: McGraw Hill.
2. Armstrong, M.A. (1979). *Basic topology* (1st ed.). New York: McGraw Hill.
3. Mendelson, B. (2009). *Introduction to topology* (3rd ed.). New York: Dover Publications.

MATH-6113 **Differential Geometry** **3(3+0)**

Differential geometry is the study of geometric properties of curves, surfaces, & their higher dimensional analogues using the methods of calculus. It has a long & rich history, &, in addition to its intrinsic mathematical value & important connections with various other branches of mathematics, it has many applications in various physical sciences, e.g., solid mechanics, computer tomography, or general relativity. Differential geometry is a vast subject. This course covers many of the basic concepts of differential geometry in the simpler context of curves & surfaces in ordinary 3-dimensional Euclidean space. The aim is to build both a solid mathematical underst&ing of the fundamental notions of differential geometry & enough visual & geometric intuition of the subject. This course is of interest to students from a variety of math, science & engineering backgrounds, & that after completing this course, the students will ready to study more advanced topics such as global properties of curves & surfaces, geometry of abstract manifolds, tensor analysis, & general relativity.

Contents

1. Space Curves
2. Arc length, tangent
3. Normal & binormal
4. Curvature & torsion of a curve
5. Tangent planes
6. The Frenet-Serret apparatus
7. Fundamental existence theorem of plane curves
8. Four vertex theorem, Isoperimetric inequality
9. Surfaces
10. First fundamental form
11. Isometry & conformal mappings
12. Curves on Surfaces, surface Area
13. Second fundamental form
14. Normal & Principle curvatures
15. Gaussian & Mean curvatures
16. Geodesics

Recommended Texts

1. Somasundaran, D. (2005). *Differential geometry* (1st ed.). New Delhi: Narosa Publishing House.
2. Pressley, A. (2001). *Elementary differential geometry* (1st ed.). New York: Springer-Verlag.

Suggested Readings

1. Wilmore, T. J. (1959). *An introduction to differential geometry* (1st ed.). Oxford Calarendon Press.
2. Weatherburn, C. E. (2016). *Differential geometry of three dimensions*. Cambridge University Press.
3. Millman, R. S., & Parker, G. D. (1977). *Elements of differential geometry*. Englewood Cliffs: Prentice Hall.

MATH-6114 Ordinary Differential Equations 3(3+0)

This course introduces the theory, solution, & application of ordinary differential equations. Topics discussed in the course include methods of solving first-order differential equations, existence & uniqueness theorems, second-order linear equations, power series solutions, higher-order linear equations, systems of equations, non-linear equations, Sturm-Liouville theory, & applications. The relationship between differential equations & linear algebra is emphasized in this course. An introduction to numerical solutions is also provided. Applications of differential equations in physics, engineering, biology, & economics are presented. The goal of this course is to provide the student with an underst&ing of the solutions & applications of ordinary differential equations. The course serves as an introduction to both nonlinear differential equations & provides a prerequisite for further study in those areas.

Contents

1. Introduction to differential equations: Preliminaries & classification of differential equations
2. Verification of solution, existence of unique solutions, introduction to initial value problems
3. Basic concepts, formation & solution of first order ordinary differential equations
4. Separable equations, linear equations, integrating factors, Exact Equations
5. Solution of nonlinear first order differential equations by substitution, Homogeneous Equations,
6. Bernoulli equation, Ricaati's equation & Clairaut equation
7. Modeling with first-order ODEs: Linear models, Nonlinear models
8. Higher order differential equations: Initial value & boundary value problems
9. Homogeneous & non-homogeneous linear higher order ODEs & their solutions, Wronskian,
10. Reduction of order, homogeneous equations with constant coefficients,
11. Nonhomogeneous equations, undetermined coefficients method, Superposition principle
12. Annihilator approach, variation of parameters, Cauchy-Euler equation,
13. Solving system of linear differential equations by elimination
14. Solution of nonlinear differential equations
15. Power series, ordinary & singular points & their types, existence of power series solutions
16. Frobenius theorem, existence of Frobenius series solutions
17. The Bessel, Modified Bessel, Legendre & Hermite equations & their solutions
18. Sturm-Liouville problems: Introduction to eigen value problem, adjoint & self-adjoint operators,
19. Self-adjoint differential equations, eigen values & eigen functions
20. Sturm-Liouville (S-L) boundary value problems, regular & singular S-L problems

Recommended Texts

1 Boyce, W. E., & Diprima, R. C. (2012). *Elementary differential equations & boundary value problems* (10^{th} ed.) USA: John Wiley & Sons.

2 Zill, D.G., & Michael, R. (2009) *Differential equations with boundary-value problems* (5^{th} ed.) New York: Brooks/Cole.

Suggested Readings

1 Arnold, V. I. (1991). *Ordinary differential equations* (3^{rd} ed.). New York: Springer.

2 Apostol, T. (1969). *Multi variable calculus & linear algebra* (2^{nd} ed.). New York: John Wiley & sons.

MATH-6115 **Real Analysis –I** **3(3+0)**

This is the first part of a two-semester course. This course covers the fundamentals of mathematical analysis: convergence of sequences & series, continuity, differentiability, Riemann integral, sequences & series of functions, uniformity, & the interchange of limit operations. It shows the utility of abstract concepts & teaches an underst&ing & construction of proofs. It develops the fundamental ideas of analysis & is aimed at developing the student's ability to describe the real line as a complete, ordered field, to use the definitions of convergence as they apply to sequences, series, & functions, to determine the continuity, differentiability & integrability of functions defined on subsets of the real line, to write solutions to problems & proofs of theorems that meet rigorous st&ards based on content, organization & coherence, argument & support, & style & mechanics, to determine the Riemann integrability of a bounded function & prove a selection of theorems concerning integration, to recognize the difference between pointwise & uniform convergence of a sequence of functions & to illustrate the effect of uniform convergence on the limit function with respect to continuity, differentiability, & integrability.

Contents

1. Number Systems: Ordered fields
2. rational, real & complex numbers
3. Archimedean property
4. supremum, infimum & completeness
5. Topology of real numbers
6. Convergence, completeness, completion of real numbers
7. Heine Borel theorem
8. Sequences & Series of Real Numbers
9. Limits of sequences, algebra of limits
10. Bolzano Weierstrass theorem, Cauchy sequences, liminf, limsup
11. limits of series, convergences tests, absolute & conditional convergence, power series
12. Continuity: Functions, continuity & compactness, existence of minimizers & maximizers
13. uniform continuity, continuity & connectedness, intermediate mean value theorem
14. monotone functions & discontinuities
15. Differentiation: Mean value theorem, L'Hopital's Rule, Taylor's theorem

Recommended Texts

1. Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4th ed.) New York: John Wiley & Sons.
2. Trench, W. F. (2013). *Introduction to real analysis* (2nd ed.). New Jersey: Prentice Hall.

Suggested Readings

1. Folland, G.B. (1999). *Real analysis* (2nd ed.). New York: John Wiley & Sons.
2. Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.) New York: McGraw-Hill.
3. Royden, H., & Fitzpatrick, P. (2010). *Real analysis* (4th ed.). New Jersey: Pearson Hall.

MATH-6116 **Algebra-II** **3(3+0)**

This course is contnuition of the course series of Algebra, which builds on the concepts learnt in Algebra I. This course is an introduction to ring theory. The philosophy of this subject is that we focus on similarities in arithmetic structure between sets (of numbers, matrices, functions or polynomials for example) which might look initially quite different but are connected by the property of being equipped with operations of addition and multiplication. Much of the activity that led to the modern formulation of ring theory took place in the first half of the 20th century. Ring theory is powerful in terms of its scope and generality, but it can be simply described as the study of systems in which addition and multiplication are possible. The objectives of the course are to introduce students to the basic ideas & methods of modern algebra & enable them to underst& the idea of a ring & an integral domain, & be aware of examples of these structures in mathematics; appreciate & be able to prove the basic results of ring theory; The topics covered include ideals, quotient rings, ring homomorphism, the Euclidean algorithm & the principal ideal domains.

Contents

1. Rings: Definition, examples. Quadratic integer rings
2. Examples of non-commutative rings
3. The Hamilton quaternions
4. Polynomial rings
5. Matrix rings. Units, zero-divisors
6. Nilpotents, idempotents. Subrings, Ideals
7. Maximal & prime Ideals. Left, right & two-sided ideals; Operations with ideals
8. The ideal generated by a set. Quotient rings. Ring homomorphism
9. The isomorphism theorems, applications
10. Finitely generated ideals
11. Rings of fractions
12. Integral Domain: The Chinese remainder theorem. Divisibility in integral domains
13. Greatest common divisor, least common multiple
14. Euclidean domains, the Euclidean algorithm, Principal ideal domains
15. Prime & irreducible elements in an integral domain
16. Gauss lemma, irreducibility criteria for polynomials

Pre-requisite: Algebra-I

Recommended Texts

1. Gallian, J. A. (2017). *Contemporary Abstract algebra* (9th ed.) New York: Brooks/Cole.
2. Malik D. S., & Mordeson J. N., & Sen M. K. (1997). *Fundamentals of abstract algebra* (1st ed.). New York: WCB/McGraw-Hill.

Suggested Readings

1. Roman, S. (2012). *Fundamentals of group theory* (1st ed.). Switzerland: Birkhäuser Basel.
2. Rose, J. (2012). *A course on group theory*. New York: Dover Publications.
3. Fraleigh, J. B. (2003). A *first course in abstract algebra* (7th ed.). New York: Pearson.

MATH-6117 **Classical Mechanics** **3(3+0)**

The purpose of this course is to provide solid underst&ing of classical mechanics & enable the students to use this underst&ing while studying courses on quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics & continuum mechanics. The course aims at familiarizing the students with the dynamics of system of particles, kinetic energy, motion of rigid body, Lagrangian & Hamiltonian formulation of mechanics. At the end of this course the students will be able to underst& the fundamental principles of classical mechanics, to master concepts in Lagrangian & Hamiltonian mechanics important to develop solid & systematic problem solving skills. To lay a solid foundation for more advanced study of classical mechanics & quantum mechanics.

Contents

1. Work, power, kinetic energy & energy principle
2. conservative force fields, conservation of energy theorem, impulse
3. Conservation of linear & angular momentum
4. Time varying mass systems (Rockets)
5. Introduction to rigid bodies
6. Translations & rotations
7. Linear & angular velocity of a rigid body about a fixed axis
8. Angular momentum for n particles
9. Rotational kinetic energy
10. Moments & products of inertia
11. Parallel & perpendicular axes theorem
12. Principal axes & principal moments of inertia. Determination of principal axes by diagonalizing the inertia matrix
13. Equimomental systems
14. Coplanar distribution
15. Rotating axes theorem
16. Euler;s dynamical equations of motion. Free rotation of a rigid body with three different principal moments, torque free motion of a symmetrical top
17. The Eulerian angles, angular velocity & kinetic energy in terms of Euler angles

Recommended Texts

1. DiBenedetto, E. (2011). *Classical mechanics: Theory & mathematical modeling*. Basel: Birkhauser.
2. Aruldhas, G. (2016). *Classical mechanics*. Dehli: PHI Private limited.

Suggested Readings

1. Spiegel, M. R. (2004). *Theoretical mechanics* (3rd ed.). Boston: Addison-Wesley Publishing Company.
2. Fowles, G. R., & Cassiday, G. L. (2005). *Analytical mechanics* (7th ed.). New York: Thomson Brooks/COLE.
3. Mir, K. L. (2007). *Theoretical mechanics*. Lahore: Ilmi Ketab Khana.

MATH-6118 **Mathematical Methods** **3(3+0)**

Mathematical methods present an applied mathematics course designed to provide the necessary analytical and numerical background for courses in astrophysics, plasma physics, fluid dynamics, electromagnetism, and radiation transfer. The main objective of this course is to provide the students with a range of mathematical methods that are essential to the solution of advanced problems encountered in the fields of applied physics & engineering. Calculation-oriented mathematics is included in all topics relevant. Systems of linear equations, Gauss-Jordan-elimination, basic matrix algebra, determinants. Limits and continuity, differensiation and integration of functions in one variable, maxima and minima, implicit differensiation and trigonometric functions, related rates, differentials and linearization, L'Hopitals rule, Newton's method and the bisection method. Riemannsums and the fundamental theorem in calculus, integral functions, definite and and indefinite integrals, basic integration techniques, substitution and partial integration, numerical integration by the rectangle and trapezium methods, improper integrals.

Contents

1. Fourier Methods: The Fourier transforms
2. Fourier analysis of the generalized functions
3. The Laplace transforms
4. Hankel transforms for the solution of PDEs & their application to boundary value problems
5. Green’s Functions & Transform Methods: Expansion for Green’s functions
6. Transform methods. Closed form Green’s functions. Perturbation Techniques
7. Perturbation methods for algebraic equations
8. Perturbation methods for differential equations
9. Variational Methods: Euler-Lagrange equations
10. Integr& involving one, two, three & n variables
11. Special cases of Euler-Lagrange’s equations
12. Necessary conditions for existence of an extremum of a functional
13. Constrained maxima & minima

Recommended Texts

1. Powers, D. L. (2005). *Boundary value problems & partial differential equations* (5th ed.). Boston: Academic Press.
2. Boyce, W. E. (2005). *Elementary differential equations* (8th ed.). New York: John Wiley & Sons.

Suggested Readings

1. Brown, J. W., & Churchil, R. V. (2006). *Fourier series & boundary value problems*. New York: McGraw Hill.
2. Snider, A. D. (2006). *Partial differential equations*. New York: Dover Publications Inc.
3. Boyce, W. E. (2005). *Elementary differential equations* (8th ed.). New York: John Wiley & Sons.
4. Krasnov, M. L., Makarenko, G. I., & Kiselev, A. I. (1985). *Problems & exercises in the calculus of variations*. USA: Imported Publications, Inc.

MATH-6119 Complex Analysis 3(3+0)

This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis & especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context. Complex Analysis is a topic that is extremely useful in many applied topics such as numerical analysis, electrical engineering, physics, chaos theory, & much more, & you will see some of these applications throughout the course. In addition, complex analysis is a subject that is, in a sense, very complete. The concept of complex differentiation is much more restrictive than that of real differentiation & as a result the corresponding theory of complex differentiable functions is a particularly nice one. Area, volume and arc length. Modeling with differential equations, first order separable and linear differential equations, Euler's method, second order linear differential equations with constant coefficients.

Contents

1. Introduction: The algebra of complex numbers
2. Geometric representation of complex numbers
3. Polar form of complex numbers
4. Powers & roots of complex numbers
5. Functions of Complex Variables
6. Limit
7. Continuity
8. Differentiable functions, the Cauchy-Riemann equations
9. Analytic functions, entire functions, harmonic functions
10. Elementary functions: The exponential, Trigonometric functions
11. Hyperbolic, Logarithmic & Inverse elementary functions
12. Complex Integrals: Contours & contour integrals, antiderivatives, independence of path
13. Cauchy-Goursat theorem, Cauchy integral formula, Lioville’s theorem, Morerea’s theorem
14. Maximum Modulus Principle
15. Series: Power series, Radius of convergence & analyticity
16. Taylor’s & Laurent’s series
17. Integration & differentiation of power series, isolated singular points

Recommended Texts

1. Mathews J. H., & Howell, R.W. (2006). *Complex analysis for mathematics & engineering* (5th ed.). Burlington: Jones & Bartlett Publication.
2. Churchill, R.V., & Brown, J.W. (2013). *Complex variables & applications* (9th ed.). New York: McGraw-Hill.

Suggested Readings

1. Remmert, R. (1998). *Theory of complex functions* (1st ed.). New York: Springer-Verlag.
2. Rudin, W. (1987). *Real & complex analysis* (3rd ed.). New York: McGraw-Hill.

MATH-6120 **Functional Analysis** **3(3+0)**

This course extends methods of linear algebra & analysis to spaces of functions, in which the interaction between algebra & analysis allows powerful methods to be developed. The course will be mathematically sophisticated & will use ideas both from linear algebra & analysis. This is a basic graduate level course that introduces the student to Functional Analysis & its applications. It starts with a review of the theory of metric spaces, the theory of Banach spaces & proceeds to develop some key theorems of functional analysis. Then continuous to linear operators in Banach & Hilbert spaces & to spectral theory of self-adjoint operators with applications to the theory of boundary value problems, & the theory of linear elliptic partial differential equations. The concept of complex differentiation is much more restrictive than that of real differentiation & as a result the corresponding theory of complex differentiable functions is a particularly nice one. Area, volume and arc length. Modeling with differential equations, first order separable and linear differential equations, Euler's method, second order linear differential equations with constant coefficients.

Contents

1. Metric Spaces
2. Convergence
3. Cauchy's sequences & examples
4. Completeness of metric space
5. Completeness proofs
6. Normed linear Spaces, Banach Spaces
7. Equivalent norms
8. Linear operators
9. Finite dimensional normed spaces
10. Continuous & bounded linear operators
11. Linear functional, Dual spaces
12. Linear operator & functional on finite dimensional Spaces
13. Inner product Spaces
14. Hilbert Spaces
15. Conjugate spaces
16. Representation of linear functional on Hilbert space

Recommended Texts

1. Kreyszig, E. (1989). *Introduction to functional analysis with applications* (1st ed.). New York: John Wiley & Sons.

Suggested Readings

1. Dunford, N., & Schwartz, J. T. (1958). *Linear operators, part-1 general theory*. New York: Interscience publishers.
2. Balakrishnan, A. V. (1981). *Applied functional analysis* (2nd ed.). New York: Springer-Verlag.
3. Conway, J. B. (1995). *A Course in functional analysis* (2nd ed.). New York: Springer-Verlag.

MATH-6121 Real Analysis –II 3(3+0)

This course is continuation of Real Analysis I, this course will continue to cover the fundamentals of real analysis, concentrating on the Riemann-Stieltjes integrals, Functions of Bounded Variation, Improper Integrals, & convergence of series. Emphasis would be on proofs of main results. The aim of this course is also to provide an accessible, reasonably paced treatment of the basic concepts & techniques of real analysis for students in these areas. This course provides greatly strengthening student's underst&ing of the results of calculus & the basis for their validity the uses of deductive reasoning, increasing the student's ability to underst& definitions, underst& proofs, analyze conjectures, find counter-examples to false statements, construct proofs of true statements & enhancing the student's mathematical communication skills.

Contents

1. The Riemann-Stieltjes Integrals
2. Definition & existence of integrals
3. Properties of integrals
4. Fundamental theorem of calculus & its applications
5. Change of variable theorem, integration by parts
6. Functions of Bounded Variation
7. Definition & examples, properties of functions of bounded variation
8. Improper Integrals: Types of improper integrals
9. Tests for convergence of improper integrals
10. Beta & gamma functions
11. Absolute & conditional convergence of improper integrals
12. Sequences & Series of Functions
13. Power series, definition of pointwise & uniform convergence
14. Uniform convergence & continuity
15. Uniform convergence & differentiation, examples of uniform convergence

Recommended Texts

1. Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4th ed.). New York: John Wiley & Sons.
2. Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.). New York: McGraw-Hill.

Suggested Readings

1. Folland, G. B. (1999). *Real analysis* (2nd ed.). New York: John Wiley & Sons.
2. Hewitt, E., & Stromberg, K. (1965). *Real & abstract analysis*. New York: Springer-Verlag Heidelberg
3. Lang, S. (1968). *Analysis I*. Boston: Addison-Wesley Publ. Co.

MATH-6122 **Numerical Analysis-I** **3(3+0)**

This course is designed to teach the students about numerical methods & their theoretical bases. The course aims at inculcating in the students the skill to apply various techniques in numerical analysis, underst& & do calculations about errors that can occur in numerical methods & underst& & be able to use the basics of matrix analysis. It is optimal to verifying numerical methods by using computer programming (MatLab, Maple, C++, etc.) This course provides greatly strengthening student's underst&ing of the results of calculus & the basis for their validity the uses of deductive reasoning, increasing the student's ability to underst& definitions, underst& proofs, analyze conjectures, find counter-examples to false statements, construct proofs of true statements & enhancing the student's mathematical communication skills.

Contents

1. Error analysis: Floating point arithmetic, Approximations & errors
2. Methods for the solution of nonlinear equations
3. Bisection method, regula-falsi method, Fixed point iteration method
4. Newton-Raphson method, secant method, error analysis for iterative methods
5. Interpolation & polynomial approximation
6. Forward, backward & centered difference formulae
7. Lagrange interpolation, Newton's divided difference formula
8. Interpolation with a cubic spline, Hermite interpolation, Least squares approximation
9. Numerical differentiation & Integration: Forward, backward & central difference formulae
10. Richardson's extrapolation, Newton-Cotes formulae, Numerical integration
11. Rectangular rule, trapezoidal rule, Simpson's 1/3 & 3/8 rules
12. Boole's & Weddle's rules, Gaussian quadrature
13. Numerical solution of a system of linear equations
14. Direct methods: Gaussian elimination method
15. Gauss-Jordan method; matrix inversion; LU-factorization

Recommended Texts

1. Gerald, C.F., & Wheatley, P.O. (2005). *Applied numerical analysis.* London: Pearson Education, Singapore.
2. Burden, R. L., Faires, J. D., & Burden, A.M. (2015). *Numerical analysis* (10^{th} ed.). Boston: Cengage Learning.

Suggested Readings

1. Philip, J. (2019). *Numerical applied computational programming with case studies* (1^{st} ed.). *New York: Apress*.
2. Khoury, R., & Harder, D.W. (2016). *Numerical methods & modelling for engineering* (1^{st} ed.). London: *Springer.*
3. Antia, H.M. (2012). *Numerical methods for scientists & engineers* (3^{rd} ed.). New York: Springer.

MATH-6123 **Number Theory** **3(3+0)**

Number theory (or arithmetic or higher arithmetic in older usage) is a branch of pure mathematics devoted primarily to the study of the integers & integer-valued functions. Integers can be considered either in themselves or as solutions to equations (Diophantine geometry). There are two subfields of number theory. One is Analytical Number Theory and other is Algebraic number theory. The focus of the course is on study of the fundamental properties of integers & develops ability to prove basic theorems. The specific objectives include study of division algorithm, prime numbers & their distributions, Diophantine equations & the theory of congruences. Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, & about unique factorisation into ideals. They will learn to calculate class numbers, & to use the theory to solve simple Diophantine equations.

Contents

1. Divisibility
2. Euclid’s theorem
3. Congruences, Elementary properties
4. Residue classes & Euler’s function
5. Linear congruence & congruence of higher degree
6. Congruences with prime moduli
7. The theorems of Fermat
8. Euler & Wilson theorem
9. Primitive roots & indices
10. Integers belonging to a given exponent
11. Composite moduli Indices
12. Quadratic Residues
13. Composite moduli
14. Legendre symbol
15. Law of quadratic reciprocity, The Jacobi symbol
16. Number-Theoretic Functions
17. Mobius function
18. The function [x]
19. Diophantine Equations
20. Equations & Fermat’s conjecture for n = 2, n = 4

Recommended Texts

1. Rosen, K.H. (2000). *Elementary number theory & its applications* (4th ed.). Boston: Addison-Wesley.
2. Apostal, T.M. (2010). *Introduction to analytic number theory* (3rd ed.). New York: Springer.

Suggested Readings

1. Leveque, W. J. (2002). *Topics in number theory*, Volumes I & II. New York: Dover Books.
2. Burton, D. M. (2007). *Elementary number theory*. New York: McGraw-Hill.

MATH-6124 Partial Differential Equations 3(3+0)

Partial Differential Equations (PDEs) are in the heart of applied mathematics & many other scientific disciplines. The beginning weeks of the course aim to develop enough familiarity & experience with the basic phenomena, approaches, & methods in solving initial/boundary value problems in the contexts of the classical prototype linear PDEs of constant coefficients: the Laplace equation, the wave equation & the heat equation. A variety of tools & methods, such as Fourier series/eigenfunction expansions, Fourier transforms, energy methods, & maximum principles will be introduced. More importantly, appropriate methods are introduced for the purpose of establishing quantitative as well as qualitative characteristic properties of solutions to each class of equations.

Contents

1. First order PDEs: Introduction, Formation of PDEs, Solutions of PDEs of first order
2. The Cauchy's problem for quasi linear first order PDEs, First order nonlinear equations
3. Special types of first order equations Second order PDEs
4. Basic concepts & definitions, Mathematical problems, Linear operator
5. Superposition, Mathematical models
6. The classical equations, The vibrating string, The vibrating membrane
7. Conduction of heat solids, Canonical forms & variable
8. PDEs of second order in two independent variables with constant & variable coefficients
9. Cauchy's problem for second order PDEs in two independent variables
10. Methods of separation of variables, Solutions of elliptic
11. Parabolic & hyperbolic PDEs in Cartesian & cylindrical coordinates
12. Laplace transform: Introduction & properties of Laplace transform
13. Transforms of elementary functions, Periodic functions, error functions
14. Dirac delta function, Inverse Laplace transform, Convolution Theorem
15. Solution of PDEs by Laplace transform, Diffusion & wave equations
16. Fourier transforms, Fourier integral representation
17. Fourier sine & cosine representation, Fourier transform pair
18. Transform of elementary functions & Dirac delta function, Finite Fourier transforms
19. Solutions of heat, Wave & Laplace equations by Fourier transforms

Recommended Texts

1. Zill, D. G., & Michael, R. (2009). *Differential equations with boundary-value problems* (5th ed.) New York: Brooks/Cole.
2. Polking, J., & Boggess, A. (2005). *Differential equations with boundary value problems* (2nd ed.). London: Pearson.

Suggested Readings

1. Wloka, J. (1987). *Partial differential equations* (1st ed.). Cambridge: Cambridge University Press.
2. Humi, M., & Miller, W. B. (1991). *Boundary value problems & partial differential equations* (1st ed.). Boston: PWS- KENT Publishing Company.

MATH-6125 **Numerical Analysis- II** **3(3+0)**

This course is designed to teach the students about numerical methods & their theoretical bases. The main purpose of this course is to learn the concepts of numerical methods in solving mathematical problems numerically & analyze the error for these methods. The students are expected to know computer programming to be able to write program for each numerical method. Knowledge of calculus & linear algebra would help in learning these methods. The students are encouraged to read certain books containing some applications of numerical methods. A variety of tools & methods, such as Fourier series/eigenfunction expansions, Fourier transforms, energy methods, & maximum principles will be introduced. More importantly, appropriate methods are introduced for the purpose of establishing quantitative as well as qualitative characteristic properties of solutions to each class of equations.

Contents

1. Difference & Differential equation
2. Formulation of difference equations
3. Solution of linear/non-linear difference equations with constant coefficients
4. Solution of homogeneous difference equations with constant coefficients
5. Solution of inhomogeneous difference equations with constant coefficients
6. The Euler method
7. The modified Euler method
8. Runge-Kutta methods
9. Predictor-corrector type methods for solving initial value problems along with convergence
10. Predictor-corrector type methods for solving initial value problems along with instability criteria
11. Runge-Kutta methods for solving initial value problems
12. Predictor-corrector type methods for solving initial value problems.

Recommended Texts

1. Gerald, C. F., & Wheatley, P.O. (2003). *Applied numerical analysis* (7th ed.). London: Pearson.
2. Balfour, A., & Beveridge, W. T. (1977). *Basic numerical analysis with FORTARAN*. New Hampshire: Heinmann Educational Books Ltd.

Suggested Readings

1. Kuo, Shan S. (1972). *Computer applications of numerical methods*. Islamabad: National Book Foundations.
2. Philip, J. (2019). *Numerical applied computational programming with case studies* (1st ed.). New York: Apress.
3. Khoury, R., & Harder, D.W. (2016). *Numerical methods & modelling for engineering* (1st ed.). London: Springer.
4. Antia, H.M. (2012). *Numerical methods for scientists & engineers* (3rd ed.). New York: Springer.

MATH-6126 **Integral Equations** **3(3+0)**

Many physical problems that are usually solved by differential equation methods can be solved more effectively by integral equation methods. This course will help students gain insight into the application of advanced mathematics & guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems. In addition, a large class of initial & boundary value problems, associated with the differential equations, can be reduced to the integral equations, whence enjoy the advantage of the above integral presentations. This course has many applications in many sciences. This course emphasizes concepts and techniques for solving integral equations from an applied mathematics perspective. Material is selected from the following topics: Volterra and Fredholm equations, Fredholm theory, the Hilbert-Schmidt theorem; Wiener-Hopf Method; Wiener-Hopf Method and partial differential equations; the Hilbert Problem and singular integral equations of Cauchy type; inverse scattering transform; and group theory. Examples are taken from fluid and solid mechanics, acoustics, quantum mechanics, and other applications.

Contents

1. Linear integral equations of the first kind
2. Linear integral equations of the second kind
3. Relationship between differential equation & Volterra integral equation
4. Neumann series
5. Fredholm Integral equation of the second kind with separable Kernels
6. Eigen values, Eigenvectors
7. Iterated functions
8. Quadrature methods
9. Least square methods
10. Homogeneous integral equations of the second kind
11. Fredholm integral equations of the first kind
12. Fredholm integral equations of the second kind
13. Abel's integral equations
14. Hilbert Schmidt theory of integral equations with symmetric Kernels
15. Regularization & filtering techniques

Recommended Texts

1. Jerri, J. (2007). *Introduction to integral equations with applications* (2nd ed.). New York: Sampling Publishing,
2. Wazwaz, A. M. (2011). *Linear & nonlinear integral equations: methods & applications*. New York: Springer.

Suggested Readings

1. Lovitt, W. V. (2005). *Linear integral equations*. New York: Dover Publications.
2. Christian, C., Dale, D., & Hamill, W. (2014). *Boundary integral equation methods & numerical solutions* (1st ed.). New York: Springer.
3. Kanwal, R. P. (1996). *Linear integral equations: theory & technique*. Boston: Birkhauser
4. Tricomi, F. G. (1985). *Integral Equations*. New York: Dover Pub.

MATH-6127 **Advance Group theory-I** **3(3+0)**

This is the first part of the two advance course series of Group Theory. This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outst&ing beauty. It takes just three simple axioms to define a group, & it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts & results of Group Theory.

Contents

1. Group of automorphisms, inner automorphisms, definition & related results
2. Characteristic & fully invariant subgroups,
3. Symmetric Groups, cyclic permutations
4. Even & odd permutations
5. The alternating groups, conjugacy classes of symmetric & alternating groups
6. Generators of symmetric & alternating groups
7. Simple groups
8. Simplicity of symmetric & alternating groups
9. Group Action on sets or G-sets
10. Orbits & stabilizer subgroups
11. Finite direct products
12. Finitely generated abelian groups
13. P-groups, Sylow's Theorems
14. Application of Sylow's Theorems
15. Linear Groups
16. Types of Linear Groups, Classical Groups

Recommended Texts

1. Rotman, J. J. (1999). *An Introduction to the theory of groups* (4th ed). New York: Springer.
2. Shah, S.K., & Shankar A. G. (2013). *Group theory.* London: Dorling Kindersley.

Suggested Readings

1. Rose, H. E. (2009). *A course on finite groups* (1st ed). New York: Springer-Verlag.
2. Fraleigh, J. B. (2003). A *first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.
3. Malik, D. S., Mordeson J. N., & Sen M. K. (1997). *Fundamentals of abstract algebra.* New York: WCB/McGraw-Hill.
4. Rose, J. A. (2012). *Course on group theory* (Revised ed.). New York: Dover Publications.

MATH-6128 **Advance Group theory-II** **3(3+0)**

This course is the continuation of the course "Advanced Group Theory-1". This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. This course covers the advanced topics in group theory such as solvable groups, Upper & Lower Central series nilpotent groups & free groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outst&ing beauty. It takes just three simple axioms to define a group, & it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts & results of Group Theory.

Contents

1. Series in groups
2. Normal series
3. Normal series & its refinement
4. Composition series
5. Equivalent composition series
6. Jordan Holder Theorem
7. Solvable groups, definition, examples & related results
8. Upper & Lower Central series
9. Nilpotent groups
10. Characterization of finite nilpotent groups
11. The Frattini subgroups, definition, examples & related results
12. Free groups, definition, examples & related results
13. Free Product, definition, examples & related results
14. Group algebras
15. Representation modules

Recommended Texts

1. Rotman, J. J. (1999). *An Introduction to the theory of groups* (4th ed). New York: Springer.
2. Shah, S.K., & Shankar A. G. (2013). *Group theory.* London: Dorling Kindersley.

Suggested Readings

1. Rose, H. E. (2009). *A course on finite groups* (1st ed). New York: Springer-Verlag.
2. Fraleigh, J. B. (2003). A *first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.
3. Malik, D. S., Mordeson J. N., & Sen M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.
4. Rose, J. A. (2012). *Course on group theory* (Revised ed.). New York: Dover Publications.

MATH-6129 **Modern Algebra-I** **3(3+0)**

The word "algebra" means many things. The word dates back about 1200 years ago to part of the title of al-Khwarizmi's book on the subject, but the subject itself goes back 4000 years ago to ancient Babylonia & Egypt. This course introduces concepts of ring theory. The main objective of this course is to prepare students for courses which require a good background in Ring theory, Ring Homomorphism, basics Theorem etc. The focus of this course is the study of ideal theory & several domains in ring theory. Homework, graded homework, class quizzes, tests & a final exam will be used to assess the Student Learning Outcomes: Upon successful completion of the course, students will be able to: Demonstrate ability to think critically by interpreting theorems & relating results to problems in other mathematical disciplines. Demonstrate ability to think critically by recognizing patterns & principles of algebra & relating them to the number system. Work effectively with others to discuss homework problems put on the board. This will be assessed through class discussions.

Contents

1. Polynomial rings
2. Division algorithm for polynomials
3. Prime elements
4. Irreducible elements
5. Euclidean domain
6. Principal ideal domain
7. Greatest common divisor
8. Prime & irreducible elements
9. Unique factorization domain
10. Factorization of polynomials over a UFD
11. Irreducibility of polynomials
12. Eisenstein's irreducibility criterion
13. Maximal ideals
14. Prime ideals
15. Primary ideals
16. Noetherian rings
17. Artinian rings

Recommended Texts

1. Gallian, J. A. (2017). *Contemporary abstract algebra* (9th ed). New York: Brooks/Cole.
2. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.

Suggested Readings

1. Roman, S. (2005). *Field theory (Graduate Texts in Mathematics)* (2nd ed.). New York: Springer.
2. Ames, D. B. (1968). *Introduction to abstract algebra. (*1st ed.). Scranton: Pennsylvania international Textbook Co.

MATH-6130 **Modern Algebra-II** **3(3+0)**

The word “algebra” means many things. The word dates back about 1200 years ago to part of the title of al-Khwarizmi’s book on the subject, but the subject itself goes back 4000 years ago to ancient Babylonia & Egypt. Modern algebra is a cornerstone of modern mathematics. This course introduces concepts of ring & group theory. The main objective of this course is to prepare students for courses which require a good background in Group Theory, Rings, Galois Theory, Symmetric group & permutation group etc. It is assumed that the students possess some mathematical maturity & are comfortable with writing proofs. After completing this course, student will be able to: Define & state some of the main concepts & theorems of Function Analysis. Apply their knowledge of subject in the investigation of examples. Prove basic proportions concerning functional analysis.

Contents

1. Finite & finitely generated Abelian groups
2. Fields
3. Finite fields
4. Field extension
5. Galois theory
6. Galois theory of equations
7. Construction with straight-edge
8. Construction with compass
9. Splitting field of polynomials
10. The Galios groups
11. Some results on finite groups
12. Symmetric group as Galois group
13. Construct able regular n-gones
14. The Galois group as permutation group

Recommended Texts

1. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.
2. Roman, S. (2005). *Field theory (Graduate Texts in Mathematics) (*2nd ed.). New York: Springer.

Suggested Readings

1. Howie, J. M. (2006). *Fields & galois theory* (2nd ed.). New York: Springer.
2. Northcott, D. D. (1973). *A first course of Homological algebra* (1st ed.). Cambridge: Cambridge University Press.
3. Jacobson, N. (1985). *Basic algebra I* (1st ed.). New York: Freeman & Co.
4. Ames, D. B. (1968). *Introduction to abstract algebra* (1st ed.). Scranton, PA: International Textbook Co.

MATH-6131 **Algebraic Topology-I** **3(3+0)**

The course gives an introduction to algebraic topology, with emphasis on the fundamental group and the singular homology groups of topological spaces. This course aims to understand some fundamental ideas in algebraic topology; to apply discrete, algebraic methods to solve topological problems; to develop some intuition for how algebraic topology relates to concrete topological problems. The primary aim of this course is to explore properties of topological spaces. We shall consider in detail examples such as surfaces. To distinguish topological spaces, we need to define topological invariants, such as the "fundamental group" or the "homology" of a space". To enable us to do this, knowledge of basic group theory & topology is essential. Some background in real analysis would also be helpful. After completing the course students can work with cell complexes and the basic notions of homotopy theory, know the construction of the fundamental group of a topological space, can use van Kampen´s theorem to calculate this group for cell complexes and know the connection between covering spaces and the fundamental group.

Contents

1. Affine spaces
2. Singular theory
3. Chain complexes
4. Homotopy invariance of homology
5. Relation between n, & H
6. Relative homology
7. The exact homology sequences.
8. Nilpotent groups
9. Homotopy theory
10. Homotopy theory of path & maps
11. Fundamental group of circles
12. Covering spaces
13. Lifting criterion
14. Loop spaces
15. Higher homotopy group.
16. Loop spaces
17. Higher homotopy group.

Recommended Texts

1. Adhikari, M. R. (2016). *Basic algebraic topology & its applications* (1st ed.). New York: Springer
2. Hatcher, A. (2001). *Algebraic topology*. Cambridge: Cambridge University Press.

Suggested Readings

1. Greenberg, M. J., & Harper, J. R. (1981). *Algebraic topology: A first course* (1st ed.). Boulder: Westview Press.
2. Croom, F. H. (1978). *Basic concept of algebraic theory.* New York: Spinger-Verlag.
3. Kosniowski, C. A. (1980). *First course in algebraic topology.* Cambridge: Cambridge University Press

MATH-6132 **Algebraic Topology-II** **3(3+0)**

This course is a continuation of Algebraic Topology-I. In this course, the objective is the study of knots, links, surfaces & higher dimensional analogs called manifolds with the underst&ing that continuous deformations do not change objects. So, a doughnut (torus) & a coffee mug are essentially the same (homeomorphic) in this course. For example, how does a creature living on a sphere tell that she is not on the plane, on the torus, or perhaps a two holed torus? Can one turn a sphere inside out without creasing it? What would it be like to live inside a three-dimensional sphere? Can one continuously deform a trefoil knot to get its mirror image? Can the wind be blowing at every point on the earth at once? Can you tell if a graph is planar? Can you tell if a knot is trivial? Is there a list of all possible two-dimensional surfaces? How about three-dimensional ones? These are some of the motivating questions for the subject. Algebraic topology attempts to answer such questions by assigning algebraic invariants such as numbers, or groups, to topological spaces. Examples include the Euler number of a surface, the Poincare index of a vector field, the genus of a torus, the fundamental group & more fancy homology groups.

Contents

1. Relative homology
2. The exact homology sequences
3. Excion theorem & application to spheres
4. Mayer Victoris sequences
5. Jordan-Brouwer separation theorem
6. Spherical complexes
7. Betti number
8. Euler characteristic
9. Cell Complexes
10. Adjunction spaces

Recommended Texts

1. Adhikari, M. R. (2016). *Basic algebraic topology & its applications* (1st ed.). New York: Springer
2. Hatcher, A. (2001). *Algebraic topology*. Cambridge: Cambridge University Press.

Suggested Readings

1. Greenberg, M. J., & Harper, J. R. (1981). *Algebraic topology: A first course* (1st ed.). Boulder: Westview Press.
2. Croom, F. H. (1978). *Basic concept of algebraic theory.* New York: Spinger-Verlag.
3. Kosniowski, C. A. (1980). *First course in algebraic topology.* Cambridge: Cambridge University Press

MATH-6133 Advanced Functional Analysis 3(3+0)

This course is intended both for continuing mathematics students & for other students using mathematics at a high level in theoretical physics, engineering & information technology, & mathematical economics. This course introduces concepts of Fundamental Theorems & Spectral Theory. On satisfying the requirements of this course, students will have the knowledge & skills to explain the fundamental concepts of functional analysis & their role in modern mathematics & applied contexts. Moreover, it demonstrate accurate & efficient use of functional analysis techniques & the capacity for mathematical reasoning through analyzing, proving & explaining concepts from functional analysis. This course will mostly deal with the analysis of unbounded operators on a Hilbert or Banach space with a particular focus on Schrodinger operators arising in quantum mechanics. All the abstract notions presented in the course will be motivated & illustrated by concrete examples. In order to be able to present some of the more interesting material, emphasis will be put on the ideas of proofs & their conceptual underst&ing rather than the rigorous verification of every little detail.

Contents

Fundamental Theorems:

1. Zorn's lemma
2. Statement of Hahn-Banach theorem for real vector spaces
3. Hahn-Banach theorem for complex vector spaces
4. Hahn-Banach theorem for normed spaces
5. Uniform boundedness theorem
6. Open mapping theorem
7. Closed graph theorem
8. Spectral Theory:
9. Spectral properties of bounded linear operations on Normed Spaces
10. Further properties of Resolvent & spectrum
11. Use of complex Analysis in spectral theory
12. Compact linear operators on Normed Spaces

Recommended Texts

1. Kreyszig, E. (1989). *Introductory functional analysis with applications* (1st ed.). New York: John Wiley.
2. Brown, A.L. (1970). *Elements of functional analysis* (1st ed.). New York: Van Nostrand & Reinhold Company.

Suggested Readings

1. Oden, J. T. (1979). *Applied functional analysis* (1st ed.). New Jersey: Prentice-Hall Inc.
2. Brown, A.L. (1970). *Elements of functional analysis* (1st ed.). New York: Van Nostrand & Reinhold Company.

MATH-6134 **Theory of Modules** **3(3+0)**

This course is an introduction to module theory, who knows something about linear algebra and ring theory. Its main aim is the derivation of the structure theory of modules over Euclidean domains. This theory is applied to obtain the structure of abelian groups and the rational canonical and Jordan normal forms of matrices. The basic facts about rings and modules are given in full generality, so that some further topics can be discussed, including projective modules and the connection between modules and representations of groups. It aims to develop the general theory of rings & then study in some detail a new concept, that of a module over a ring. The theory of rings & module is key to many more advanced algebra courses. This subject presents the foundational material for the last of the basic algebraic structure pervading contemporary pure mathematics, namely fields & modules. The basic definitions & elementary results are given, followed by two important applications of the theory. This course introduces concepts of modules. The main objective of this course is to prepare students for courses which require a good background in Modules Theory, Primary component & Invariance Theorem etc.

Contents

1. Elementary notions & examples
2. Modules, sub modules, Quotient modules
3. Finitely generated & cyclic modules, Exact sequences
4. Elementary notions of homological algebra
5. Noetherian rings & modules
6. Artinian rings & modules, Radicals
7. Semisimple rings & modules
8. Tensor product of modules
9. Bimodules
10. Algebra & coalgebra
11. Torsion module
12. Primary components
13. Invariance theorem

Recommended Texts

1. Wang, F., & Kim, H. (2016). *Foundations of commutative rings & their modules* (1st ed.). New York: Springer.
2. Berrick, A. J., & Keating, M. E. (2000). *An introduction to rings & modules: With K-Theory in View* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Hartley, B., & Hawkes, T. O. (1980). *Rings, modules & linear algebra (*1st ed.). London: Chapman & Hall.
2. Herstein I. N. (1995). *Topics in algebra with application* (3rd ed.). New York: Books/Cole.
3. Jacobson, N. (1989). *Basic algebra* (2nd ed.). Colorado: Freeman
4. Blyth, T. S. (1977). *Module theory* (1st ed.). Oxford: Oxford University Press.

MATH-6135 **Astronomy-I** **3(3+0)**

All Physics & Astronomy courses are expected to incorporate critical thinking abilities, quantitative skills & communication skills as core objectives in their course material & course work. They will also be required to demonstrate knowledge, underst&ing & use of the principles of physics &/or astronomy. In addition, there are objectives specific to Physics, Mathematics & Astronomy discipline courses. Both our overall & course-specific learning objectives are listed below. Students are required to demonstrate: (1) Knowledge, underst&ing & use of the principles of physics &/or astronomy. (2) Ability to use reasoning & logic to define a problem in terms of principles of physics. (3) Ability to use mathematics & computer applications to solve physics &/or astronomy problems. (4) Ability to design &/or conduct experiments &/or observations using principles of physics &/or astronomy & physics or astronomical instrumentation. (5) Ability to properly analyze & interpret data & experimental uncertainty in order to make meaningful comparisons between experimental measurements or observation & theory

Contents

1. Introduction to Astronomy
2. The great & small circles
3. Spherical angle & spherical triangle
4. Applications to the Earth
5. Longitude & latitude
6. Horizontal & equatorial systems of coordinates
7. Observer's meridian
8. Diurnal motion
9. Circumpolar stars
10. Right ascension
11. The equation of time
12. Basics of spherical trigonometry
13. The celestial sphere

Recommended Texts

1. Roy, A. E. (1982). *Astronomy: Principles & practice* (1st ed.). Bristor: London: Adam Hilger Ltd.
2. Woolard, E. W., & Clemence, G. M. (1966). *Spherical astronomy* (1st ed.). Boston: Academic Press.

Suggested Readings

1. Smart, W. M. (1977). *Textbook on spherical astronomy* (1st ed.). Cambridge: Cambridge University Press.

MATH-6136 **Astronomy-II** **3(3+0)**

An objective of the Course is to provide new research students in astronomy with an introduction & overview to research topics of major current & future interest. This will allow students to acquire some underst&ing & appreciation of research fields in addition to those they will be investigating as part of their graduate studies. They will also be required to demonstrate knowledge, underst&ing & use of the principles of physics &/or astronomy. In addition, there are objectives specific to Physics, Mathematics & Astronomy discipline courses. Another objective is to provide new students with the opportunity to interact with each other, & also with lecturers who are leaders in their research field. As well as providing an introduction to astronomy research, the Course includes talks on career advice, public engagement activities in astronomy (of particular importance due to the popularity of astronomy with the general public, including children), unconscious bias in science & gender issues.

Contents

1. Introduction to celestial navigation on earth
2. Celestial sphere
3. Time-keeping system
4. Refraction
5. Parallax & triangulation
6. Aberration
7. Precession
8. Nutation
9. Tropical measurements
10. Magnitude systems
11. Naked Eye Observations
12. Observational techniques
13. Optics & telescopes
14. Radio telescopes & Doppler imaging

Recommended Texts

1. Roy, A. E. (1982). *Astronomy: Principles & practice* (1st ed.). Bristor: London: Adam Hilger Ltd.
2. Woolard, E. W., & Clemence, G. M. (1966). *Spherical astronomy* (1st ed.). Boston: Academic Press.

Suggested Readings

1. Smart, W. M. (1977). *Textbook on spherical astronomy* (1st ed.). Cambridge: Cambridge University Press.

MATH-6137 **Electromagnetism-I** **3(3+0)**

Electromagnetism is a branch of physics involving the study of the electromagnetic force, a type of physical interaction that occurs between electrically charged particles. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of coulomb's law, magnetic shells, conductivity & current density vector to flows.

Contents

1. Electrostatics: Coulomb's law
2. Electric field & potential. lines of force & equipotential surfaces
3. Gauss's law & deduction
4. Conductor condensers
5. Dipoles, forces dipoles
6. Dielectrics, polarization & apparent charges
7. Electric displacement
8. Energy of the field, minimum energy
9. Magnetostatic field
10. The magnetostatic law of force, magnetic shells
11. Force on magnetic doublets
12. Magnetic induction, paradia & magnetism
13. Steady & slowly varying currents
14. Electric current
15. Linear conductors
16. Conductivity
17. Resistance
18. Kirchoff's laws
19. Heat production
20. Current density vector
21. Magnetic field of straight & circular current
22. Magnetic flux

Recommended Texts

1. Ferraro, V. C. A. (1956). *Electromagnetic theory* (Revised ed.). London: The Athlon Press
2. Reitz, J. R., Milford, F. J., & Christy, R. W. (1960). *Foundations of electromagnetic theory* (3rd ed.). Boston: Addison-Wesley.

Suggested Readings

1. Pugh, M. E. (196). *Principles of* e*lectricity & magnetism* (1st ed.). Boston: Addison-Wesley.

MATH-6138 Electromagnetism-II 3(3+0)

This course is the continuation of the course Electromagnetism-I. The classical (non-quantum) theory of electromagnetism was first published by James Clerk Maxwell in his 1873 textbook A Treatise on Electricity and Magnetism. A host of scientists during the nineteenth century carried out the work that ultimately led to Maxwell's electromagnetism equations, which is still considered one of the triumphs of classical physics. Maxwell's description of electromagnetism, which demonstrates that electricity and magnetism are different aspects of a unified electromagnetic field, holds true today. Electromagnetism is a branch of physics involving the study of the electromagnetic force, a type of physical interaction that occurs between electrically charged particles. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of simple introduction to Legendre polynomials, method of images, images in a plane, images with spheres & cylinders.

Contents

1. Vector potential
2. Forces on a circuit in magnetic field
3. Magnetic field energy, Law of electromagnetic induction
4. Co-efficient of self & mutual induction
5. Alternating current & simple I.C.R circuits in series & parallel
6. Power factor, the equations of electromagnetism
7. Maxwell’s equations in free space & material media
8. Solution of Maxwell’s equations
9. Plane electromagnetic waves in homogeneous & isotropic media
10. Reflection & refraction of plane waves
11. Wave guides Laplace’ equation in plane, Polar & cylindrical coordinates
12. Simple introduction to Legendre polynomials
13. Method of images, images in a plane
14. Images with spheres & cylinders

Pre-requisite: Electromagnetism-I

Recommended Texts

1. Ferraro, V. C. A. (1956). *Electromagnetic theory* (Revised ed.). London: The Athlon Press
2. Reitz, J. R., Milford, F. J., & Christy, R. W. (1960). *Foundations of electromagnetic theory* (3rd ed.). Boston: Addison-Wesley.

Suggested Readings

1. Pugh, M. E. (196). *Principles of electricity & magnetism* (1st ed.). Boston: Addison-Wesley.

MATH-6139 Fluid Mechanics-I 3(3+0)

This course is the first part of the core level course on fluid mechanics. Fluid mechanics is the branch of physics concerned with the mechanics of fluids (liquids, gases, & plasmas) & the forces on them. It has applications in a wide range of disciplines, including mechanical, civil, chemical & biomedical engineering, geophysics, oceanography, meteorology, astrophysics, & biology. The course of fluid mechanics is introducing fundamental aspects of fluid flow behavior. Students will learn properties of Newtonian fluids; apply concepts of mass, momentum & energy conservation to flows. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of simple introduction to Legendre polynomials, method of images, images in a plane, images with spheres & cylinders.

Contents

1. Introduction: Definition of Fluid, basics equations
2. Methods of analysis, dimensions & units. Fundamental concepts
3. Fluid as a continuum, velocity field, stress field, viscosity, surface tension, description & classification of fluid motions
4. Fluid Statics: The basic equation of fluid static
5. The st&ard atmosphere
6. Pressure variation in a static fluid
7. Fluid in rigid body motion. Basic equation in integral form for a control volume
8. Basic laws for a system
9. Relation of derivatives to the control volume formulation
10. Conservation of mass
11. Momentum equation for inertial control volume
12. Momentum equation for control volume with rectilinear acceleration
13. Momentum equation for control volume with arbitrary acceleration
14. The angular momentum principle
15. The first law of thermodynamics

Recommended Texts

1. Fox, R. W., & McDonald, A. T. (2004). *Introduction to fluid mechanics* (6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: Mc. Graw Hill.

Suggested Readings

1. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). Montana: Winston Publisher.
2. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey Son Publisher.
3. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Oxford: Butterworh Heinemann Publisher.

MATH-6140 **Fluid Mechanics-II** **3(3+0)**

This course is the seconed part of the core level course on fluid mechanics. Fluid mechanics is concerned with the mechanics of fluids (liquids, gases, & plasmas) & the forces on them. This course covers properties of fluids, laws of fluid mechanics & energy relationships for incompressible fluids Studies flow in closed conduits, including pressure loss, flow measurement, pipe sizing & pump Selection, momentum equation for frictionless flow, Euler's equations, Bernoulli equation-Integration of Euler's equation, laminar flow & Boundary layers. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of simple introduction to Legendre polynomials, method of images, images in a plane, images with spheres & cylinders.

Contents

1. Incompressible inviscid flow
2. Momentum equation for frictionless flow
3. Euler's equations
4. Euler's equations in streamline coordinates
5. Bernoulli equation- Integration of Euler's equation along a streamline for steady flow
6. Relation between first law of thermodynamics & the Bernoulli equation
7. Unsteady Bernoulli equation-Integration of Euler's equation along a streamline
8. Irrotational flow, internal incompressible viscous flow
9. Fully developed laminar flow
10. Fully developed laminar flow between infinite parallel plates
11. Fully developed laminar flow in a pipe
12. Part-B Flow in pipes & ducts
13. Shear stress distribution in fully developed pipe flow
14. Turbulent velocity profiles in fully developed pipe flow
15. Energy consideration in pipe flow

Recommended Texts

1. Fox, R. W., & McDonald, A. T. (2004). *Introduction to fluid mechanics* (6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: Mc. Graw Hill.

Suggested Readings

1. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey Son Publisher.
2. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Oxford: Butterworh Heinemann Publisher.
3. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). Montana: Winston Publisher.

MATH-6141 **Operations Research-I** **3(3+0)**

This course is the 1st part of the course series on operation research. Operations research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. Operations Research studies analysis and planning of complex systems. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real-life decision-making problems. The purpose of the course is to provide students with the concepts and tools to help them understand the operations research and mathematical modeling methods and to understand different application areas of operations research like transportation problem, assignment model, sequencing models, dynamic programming, game theory, replacement models & inventory models.

Contents

1. Linear Programming
2. Formulation & graphical solution
3. Simplex method, M-technique
4. Two-phase technique
5. Special cases
6. Sensitivity analysis
7. The dual problem
8. Primal dual relationship
9. The dual simplex method
10. Sensitivity
11. Post optimal analysis
12. Transportation model
13. Northwest corner
14. Least cost
15. Vogel's approximation methods
16. The method of multipliers
17. The assignment models
18. The transhipment model
19. Network minimization
20. Shortest route algorithms for variables

Recommended Texts

1. Hamdy, A. T. (2006). *Operations research an introduction* (6th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1st ed.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research*: *A practical introduction* (1st ed.). North Holland: CRC Press
2. Ravindran, A. R. (2008). *Operations research application*s (1st ed.). North Holland: CRC Press.

MATH-6142 **Operations Research-II** **3(3+0)**

Operations research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. Disciplines that are similar to, or overlap with, operations research include statistical analysis, management science, game theory, optimization theory, artificial intelligence & network analysis. All of these techniques have the goal of solving complex problems & improving quantitative decisions. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real-life decision-making problems. Students would be able to identify & develop complecated operational research modals from the verbal description of the real system. The underst&ing of the mathematical tools that are needed to solve optimization problems would be increased. They would be analyzing the results & propose the theoretical language underst&able to decision making processes in Management Engineering.

Contents

1. Algorithm for cyclic network
2. Maximal flow problems
3. Matrix definition of LP- problems
4. Revised simplex methods
5. Bounded variables decompositions algorithm
6. Parametric linear programming
7. Application of integer programming
8. Cutting plane algorithm
9. Mixed fractional cut algorithm
10. Branch methods
11. Bound methods
12. Zero-one implicit enumeration

Recommended Texts

1. Hamdy, A. T. (2006). *Operations research an introduction* (6th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1st ed.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research*: *A practical introduction* (1st ed.). North Holland: CRC Press.
2. Hamdy, A. T. (2006). *Operations research an introduction* (6th ed.). New York: Macmillan.

MATH-6143 **Quantum Mechanics–I** **3(3+0)**

This course is the first part of a two-course sequence., which covered most of the basic topics in quantum mechanics, including perturbation theory, operator techniques, and the addition of angular momentum. Quantum mechanics (QM; also known as quantum physics, quantum theory, the wave mechanical model & matrix mechanics), part of quantum field theory, is a fundamental theory in physics. It describes physical properties of nature on an atomic scale. Classical physics, the description of physics that existed before the theory of relativity & quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic & subatomic) scales, for which classical mechanics is insufficient. This course will introduce Dirac's braket formulation of quantum mechanics & make students familiar with various approximation methods applied to atomic, nuclear & solid-state physics, & to scattering.

Contents

1. Inadequacy of classical mechanics
2. Black body radiation, photoelectric effect
3. Compton effect
4. Bohr’s theory of atomic structure
5. Wave-particle duality
6. The de-Broglie postulate
7. The uncertainty principle
8. Uncertainty of position
9. Momentum
10. Statement & proof of the uncertainty principle
11. Energy-time uncertainty
12. Eigenvalues & eigen functions
13. Operators & eigen functions
14. Linear operators
15. Operator formalism in quantum mechanics
16. Orthonormal systems
17. Hermitian operators & their properties,
18. Simultaneous eigen functions
19. Parity operators, postulates of quantum mechanics

Recommended Texts

1. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New South Wales: George Allen & Unwin.
2. Powell, T. L., & Crasemann, B. (1961). *Quantum mechanics* (1st ed.). Boston: Addison Wesley.

Suggested Readings

1. Merzdacker, E. (1988). *Quantum mechanics* (1st ed.). New York: John Wiley.
2. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New South Wales: George Allen & Unwin.

MATH-6144 **Quantum Mechanics–II** **3(3+0)**

This course is the second part of a two-course sequence. The primary goal of this course is to develop an understanding of some of the more advanced topics and techniques used in quantum mechanics. Most of this material will be essential for graduate research in many areas of physics, such as quantum optics, astrophysics, and atmospheric physics. This course will provide the necessary knowledge and skills to apply advanced techniques in quantum mechanics throughout the students' careers. Quantum mechanics (QM; also known as quantum physics, quantum theory, the wave mechanical model & matrix mechanics), part of quantum field theory, is a fundamental theory in physics. It describes physical properties of nature on an atomic scale. Classical physics, the description of physics that existed before the theory of relativity & quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic & subatomic) scales, for which classical mechanics is insufficient. This course is continuation of Quantum Mechanics-I & cover more advance topics.

Contents

1. Motion in three dimensions
2. Angular momentum
3. Commutation relations between components of angular momentum
4. Representation in spherical polar coordinates
5. Simultaneous Eigen functions of Lz & L2
6. Spherically symmetric potential
7. The hydrogen atom
8. Scattering Theory
9. The scattering cross-section
10. Scattering amplitude
11. Scattering equation
12. Born approximation
13. Partial wave analysis
14. Perturbation Theory
15. Time independent perturbation of non-degenerate & degenerate cases
16. Time-dependent perturbations
17. Identical Particle
18. Symmetric & anti-symmetric Eigen function
19. The Pauli exclusion principle.

Recommended Texts

1. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New South Wales: George Allen & Unwin.
2. Powell, T. L., & Crasemann, B. (1961). *Quantum mechanics* (1st ed.). Boston: Addison Wesley.

Suggested Readings

1. Merzdacker, E. (1988). *Quantum mechanics* (1st ed.). New York: John Wiley.
2. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New South Wales: George Allen & Unwin.

MATH- 6145 Analytical Dynamics 3(3+0)

In classical mechanics, analytical dynamics, or more briefly dynamics, is concerned with the relationship between motion of bodies & its causes, namely the forces acting on the bodies & the properties of the bodies, particularly mass & moment of inertia. Analytical dynamics develops Newtonian mechanics to the stage where powerful mathematical techniques can be used to determine the behavior of many physical systems. The mathematical framework also plays a role in the formulation of modern quantum & relativity theories. Classical physics, the description of physics that existed before the theory of relativity & quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic & subatomic) scales, for which classical mechanics is insufficient. This course is continuation of Quantum Mechanics-I & cover more advance topics.

Contents

1. Generalized coordinates
2. Constraints
3. Degree of freedom
4. D'Alembert principle
5. Holonomic & non-Holonomic systems, Hamilton's principle
6. Derivation of Lagrange equation from Hamilton's principle
7. Derivation of Hamilton's equation from a variational principle
8. Equations & Examples of Gauge transformations
9. Equations & examples of canonical transformations
10. Orthogonal Point transformations
11. The Principle of Least Action
12. Applications of Hamilton's equation to central force problems
13. Applications to Harmonic oscillator
14. Hamiltonian formulism
15. Lagrange bracket & Poisson brackets with application
16. The Hamilton Jacobi theory, Hamilton Jacobi Theorem
17. The Hamilton Jacobi equation for Hamilton characteristic functions
18. Bilinear co-variant
19. Quasi coordinates

Recommended Texts

1. Greenwood, D. T. (1965). *Classical dynamics*. New Jersey: Prentice-Hall, Inc.
2. Aruldhas, G. (2016). *Classical mechanics*. New Dehli: PHI Private Limited.
3. Chorlton, F. (1983). *Textbook of dynamics*. Cambridge: E. Horwood.

Suggested Readings

1. Woodhouse, N. M. J. (2009). *Introduction to analytical dynamics* (2nd ed.). New York: Springer-Verlag.
2. Chester, W. (1979). *Mechanics*. London: New South Wales: George Allen & Unwin Ltd.

MATH-6146 **Special Relativity** **3(3+0)**

This course introduces the basic ideas and equations of Einstein's Special Theory of Relativity to understand the physics of Lorentz contraction, time dilation, the "twin paradox", and $E=mc^2$. Calculus Vector transformations Tensors for GTR to understand why we need these two theories. For that see the problems with Galilean transformation & equivalence of inertial & gravitational mass. The most important thing to study SR is to accept geometry as the concept behind it. The math is not difficult; it's the way of thinking you have to adopt. Draw space time diagrams, something to transform to another frame of reference (Lorentz transforms are available). Keep in mind that the view in the other reference frame is just a different view of the same situation that nothing really has changed, even if it looks different on Euclidean paper.

Contents

1. Historical background
2. Fundamental concepts of special theory of relativity
3. Galilean transformations,
4. Lorentz transformations (for motion along one axis)
5. Length contraction
6. Time dilation
7. Simultaneity
8. Velocity addition formulae.3-dimensional
9. Lorentz transformations
10. Introduction to 4-vector formalism
11. Lorentz transformations in the 4-vector formalism
12. The Lorentz groups
13. The Poincare groups
14. Introduction to classical mechanics
15. Minkowski space-time & null cone
16. 4-velocity & 4-momentum & 4-force
17. Application of special relativity to Doppler shift & Compton effect

Recommended Texts

1. Qadir, A. (1989). *An introduction to the special relativity theory* (1st ed.). Singapore: World Scientific.
2. Sardesai, P. L. (2008). *A primer of special relativity* (2nd ed.). Delhi: Offset.

Suggested Readings

1. Resnick, R. (1968). *Introduction to special relativity*. New York: Wiley.
2. D’Inverno, R. (1992). *Introducing Einstein’s relativity* (1st ed.). Oxford: Oxford University Press.

MATH-6147 Numerical Solution of Partial Differential Equations 3(3+0)

This course addresses post graduate students of all fields who are interested in numerical methods for partial differential equations, with focus on a rigorous mathematical basis. Many modern & efficient approaches are presented, after fundamentals of numerical approximation are established. Of particular focus are a qualitative underst&ing of the considered partial differential equation, fundamentals of finite difference, finite volume, finite element, & spectral methods, & important concepts such as stability, convergence, & error analysis.Students who have successfully taken this module should be aware of the issues around the discretization of several different types of PDEs, have a knowledge of the finite element & finite difference methods that are used for discretizing, be able to discretise an elliptic partial differential equation using finite element & finite difference methods, carry out stability & error analysis for the discrete approximation to elliptic, parabolic & hyperbolic equations in certain domains. Students are able to solve following problems: advection equation, heat equation, wave equation, Airy equation, convection-diffusion problems, KdV equation, hyperbolic conservation laws, Poisson equation, Stokes problem, Navier-Stokes equations, interface problems.

Contents

1. Finite-Difference Formulae
2. Parabolic Equations
3. Finite difference methods
4. Convergence analysis
5. Stability analysis
6. Parabolic Equations
7. Alternative derivation of difference equations
8. Miscellaneous topics,
9. Hyperbolic equations
10. Characteristics,
11. Elliptic equations
12. Systematic iterative methods.

Recommended Texts

1. Morton, K. W., & Mayers, D. F. (2005). *Numerical solution of partial differential equations: An introduction* (2nd ed.). Cambridge: Cambridge University Press.
2. Bertoluzza, S., Falletta, S., Russo, G., & Chu, C. W. (1986). *Numerical solution of partial differential equations* (1st ed.). Basel: Birkhauser.

Suggested Readings

1. Ames, W. F. (1992). *Numerical methods for partial differential equations* (3rd ed.). New York: Academic Press.
2. Smith, G. D. (1986). *Numerical solution of partial differential equations: Finite difference Methods* (3rd ed.). Oxford: Oxford University Press.

MATH-6148 Elastic Theory 3(3+0)

This course is an introduction to the principal concepts and theory of elasticity. The course is intended to provide basic knowledge of Analysis of stress and strain; equilibrium; compatibility; elastic stress-strain relations; material symmetries. Torsion and bending of bars. Elasticity theory is the mathematical framework which describes such deformation. By elastic, we mean that the material rebounds to its original shape after the forces on it are removed; a rubber eraser is a good example of an elastic material. The objectives of this course are to introduce to the students the analysis of linear elastic solids under mechanical & thermal loads, to introduce theoretical fundamentals & to improve the ability to use the principles of theory of elasticity in engineering problems. Students who successfully complete the course should be expert in using indicial notion, Cartesian tensor analysis, analysis of stress & deformation, basic filed equations of linear elastic solids & to formulate solution strategies of various boundary value problems.

Contents

1. Cartesian tensors
2. Analysis of stress
3. Analysis of strain
4. Generalized Hook's law
5. Crystalline structure
6. Point groups of crystals
7. Reduction in the number of elastic moduli due to crystal symmetry
8. Equations of equilibrium
9. Boundary conditions
10. Compatibility equations
11. Plane stress
12. Plane strain problems
13. Two dimensional problems in rectangular coordinates
14. Two dimensional problems in polar coordinates
15. Torsion of rods
16. Torsion of beams

Recommended Texts

1. Sokolinikoff, I. S. (1956). *Mathematical theory of elasticity* (2nd ed.). New York: McGraw Hill.
2. Dieulesaint, E., & Royer, D. (1974). *Elastic waves in solids* (1st ed.). New York: Wiley.

Suggested Readings

1. Funk, Y. C. (1965). *Foundations of solid mechanics (*1st ed.). New Jersey: Prentice – Hall.
2. Sadd, N. H. (2005). *Theory applications & numeric.* New York: Elsevier.
3. Boresi, A. P. (2000). *Elasticity in engineering mechanics.* New York*:* Wiley.

MATH-6149 **History of Mathematics** **2(2+0)**

This course is designed to provide the historical background to some of the mathematics familiar to students. This course is a survey of the historical development of mathematics. The emphasis will be on mathematical concepts, problem solving, and pedagogy from a historical perspective. In this course, we will explore some major themes in mathematics calculation, number, geometry, algebra, infinity, formalisms & their historical developments in various civilizations. We will see how the earlier civilizations influenced or failed to influence later ones & how the conceps evolved in these various civilizations. The aims of teaching & learning mathematics are to encourage & enable students to understand & be able to use the language, symbols & notation of mathematics, develop mathematical curiosity & use inductive & deductive reasoning when solving problems. Students will demonstrate their knowledge of basic historical facts; they will demonstrate understanding of the development of mathematics and mathematical thought.

Contents

1. History of Numerations
2. Egyptian
3. Babylonian
4. Hindu contributions
5. Arabic contributions
6. Algebra: Including the contributions of Al-Khwarzmi
7. Algebra: Including the contributions of Ibn Kura
8. History of Geometry
9. History of Euclud's elements
10. History of Analysis
11. The Calculus: Newton
12. The Calculus: Leibniz
13. The Calculus: Gauss
14. The contributions of Bernoulli brothers
15. The Twentieth Century Mathematics

Recommended Texts

1. Boyer, B., & Mersbach, U. V. (1989). *The history of mathematics* (2nd ed.). San Francisco: Jossey-Bass.
2. Berlinghoff, W. P., & Gouvea, F. Q. (2004). *Math through the ages: A gentle history for teachers & others* (Expanded ed.). London: Oxton House & MAA.

Suggested Readings

1. Burton, D. M. (2011). *The history of mathematics: An introduction* (7th ed.). New York: McGraw-Hill.
2. Katz, V. J. (2009). *A history of mathematics, an introduction* (3rd ed.). New York: Addison-Wesley.
3. Dunham, W. (1990). *Journey through genius: The great theorems of mathematics*. London: Penguin Pub.

MATH-6150 **Heat Transfer** **3(3+0)**

Heat transfer is a discipline of thermal engineering that concerns the generation, use, conversion, & exchange of thermal energy (heat) between physical systems. Heat transfer is classified into various mechanisms, such as thermal conduction, thermal convection, thermal radiation, & transfer of energy by phase changes. The objectives of heat transfer include the following: Insulation, wherein across a finite temperature difference between the system & its surrounding, the engineer seeks to reduce the heat transfer as much as possible. The learning outcomes of this course are: to explain the basics of heat transfer, to explain the importance of heat transfer, to define the concept of boiling & condensation, to define the concept of heat exchangers, to explain heat transfer by conduction, to explain the Fourier heat conduction law, to define thermal conductivity coefficient & diffusion coefficient, to explain heat transfer with convection, to explain Newton's law, to explain free transport phenomenon, to explain the forced convection, to explain heat transfer by radiation.

Contents

1. Steady-State Conduction-One Dimension
2. Steady-State Conduction-Multiples Dimensions
3. Unsteady-State Conduction,
4. Principles of Convection
5. Empirical & practical Relations
6. Forced –Convection Heat Transfer
7. Natural Convection Systems
8. Radiation Heat Transfer

Recommended Texts

1. Holman, J. P. (1996). *Heat transfer* (8th ed.). New York: McGraw Hill.
2. Kays, W. M., & Crawford, M. E. (1993). *Convective heat & mass transfer* (*3*rd ed.). New York: McGraw Hill.

Suggested Readings

1. Incropera, F. P., & Dewitt, D. P. (1985). *Fundamentals of heat & mass transfer* (2nd ed.). New York: Wiley.
2. Cenegel, Y., & Ghajar, A. J. (2015). *Heat & mass transfer: Fundamentals & applications* (5th ed.). New York: Mc-Graw Hill.
3. Lienhar IV, J. H., & Lienhar V, J. H. (2019). *A heat transfer textbook* (5th ed.). New York: Dover Publications.
4. Incropera, F. P. (2006). *Fundamentals of heat & mass transfer* (6th ed.). New York: John Wiley & Sons.

MATH-6151 **Bio-Mathematics** **3(3+0)**

Mathematical & theoretical biology is a branch of biology which employs theoretical analysis, mathematical models & abstractions of the living organisms to investigate the principles that govern the structure, development & behavior of the systems, as opposed to experimental biology which deals with the conduction of experiments to prove & validate the scientific theories. The objective of this course is to meet the current & future needs for the interaction between mathematics & biological sciences. Mathematical modeling is being applied in every major discipline in the biomedical sciences. A very different applications, & surprisingly successful, is in psychology, modeling of various human interactions, blood flow & functioning of different organs in human body. Mathematics may be divided into the broad categories of analysis (calculus), algebra, geometry & logic. This subject fit largely into the calculus category & follows on from material you will have learned in first year & from other related courses you may have taken, although algebra & areas will also be involved. This course is very useful for those majoring in Applied Mathematics, those planning to teach, or those students of Mathematics who are interested in the application of mathematical techniques to real-world problem solving.

Contents

1. An introduction to the use of continuous differential equations in the biological sciences
2. An introduction to the use of discrete differential equations in the biological sciences
3. Single species
4. Interacting population dynamics
5. Modeling infectious & dynamic diseases
6. Modeling infectious diseases
7. Modeling dynamic diseases
8. Regulation of cell function
9. Molecular interactions
10. Neural & biological oscillators
11. Introduction to biological pattern formation
12. Mathematical tools such as phase portraits
13. Bifurcation diagrams
14. Perturbation theory
15. Parameter estimation techniques
16. Interpretation of biological models

Recommended Texts

1. Murray, J. D. (2001). *Mathematical biology.* New York: Springer-Verlag.
2. Britton, N. F. (2003). *Essential mathematical biology*. New York: Springer- Verlag

Suggested Readings

1. Keener, J., & Sneyd, J. (1998). *Mathematical physiology*. New York: Springer.
2. Edelstein-Keshet, L. (1988). *Mathematical models in biology*. New York: R&om House.

MATH-6152 **Theory of Automata** **3(3+0)**

Automata theory is the study of abstract machines & automata, as well as the computational problems that can be solved using them. It is a theory in theoretical computer science. The word automata (the plural of automaton) comes from the Greek word αὐτόματα, which means "self-making". The major objective of automata theory is to develop methods by which computer scientists can describe & analyze the dynamic behavior of discrete systems, in which signals are sampled periodically. ... Inputs: assumed to be sequences of symbols selected from a finite set I of input signals. The aim is to introduce to the students to the foundations of computability theory. Other objectives include the application of mathematical techniques & logical reasoning to important problems, & to develop a strong background in reasoning about finite automata & formal languages. At the end of the course the students should be able to: define the notion of countable & uncountable set, define the various categories of languages & grammars, define various categories of automata, define the notion of computability & decidability, & reduce a problem to another(when possible) to develop proofs of decidability/undecidibility. The course introduces some fundamental concepts in automata theory & formal languages including grammar, finite automaton, regular expression, formal language, pushdown automaton, & Turing machine. Not only do they form basic models of computation, they are also the foundation of many branches of computer science, e.g. compilers, software engineering, concurrent systems, etc.

Contents

1. Regular expressions
2. Regular Languages
3. Finite Automata
4. Context-free Grammars
5. Context-free languages
6. Push down automata
7. Decision Problems
8. Parsing
9. Turing Machines

Recommended Texts

1. Martin, J. C. (2010). *Introduction to languages & theory of computation* (4th ed.). New York: Mc Graw Hill.
2. Michael S. (2013). *Introduction to the theory of computation* (3rd ed.). New York: Cengage Learning.

Suggested Readings

1. Cohen, D. I. A. (1996). *Introduction to computer theory* (2nd ed.). New York: Wiley.
2. Linz, P. (2017). *Introduction to formal languages & automata* (6th ed.). New York: Jones & Barlett.

MATH-6153 **Measure Theory** **3(3+0)**

The objectives of the course are to introduce the concepts of measure & integral with respect to a measure, to show their basic properties, to provide a basis for further studies in analysis, probability, & dynamical Systems, to construct Lebesgue's measure & learn the theory of Lebesgue integrals on real line & in n-dimensional Euclidean space. The goal of the course is to develop the understanding of basic concepts of measure and integration theory. As measure theory is a part of the basic curriculum since it is crucial for understanding the theoretical basis of probablity and statistics, so it is intended to develop understanding of the theory based on examples of application. After the course the students will know & understand the basic concepts of measure theory & the theory of Lebsgue integration. The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs himself, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

1. Introduction to Lebesgue measure
2. Outer measure
3. Properties of outer measure
4. Further properties of outer measure
5. Measurable sets
6. Properties of measurable sets
7. Non measurable sets
8. Measurable functions
9. Properties of measurable functions
10. Convergence of sequences of measurable functions
11. Lebesuge integration, introduction
12. Lebesgue integrals of simple
13. Bounded functions
14. Lebesgue integrals of nonnegative functions

Recommended Texts

1. Roydon, H. L., & Fitzpatrick, P. M. (2017). *Real analysis* (4^{th}ed.). New York: Collier Macmillan Co.
2. Barra, G. D. (1981). *Measure theory & integration* (1^{st} ed.). Ellis: Harwood Ltd.

Suggested Readings

1. Rudin, W. (1987). *Real & complex analysis*, (3^{rd}ed.). New York: McGraw Hill Book Company.
2. Bartle, R.G. (1995). *The elements of integration & Lebesgue measure* (1^{st} ed.). Wiley-Interscience.
3. Halmos, P. R. (1975). *Measure theory* (1^{st} ed.). New York: Springer.

MATH-6154 **Special Functions** **3(3+0)**

Special functions are particular mathematical functions that have more or less established names & notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications. The term is defined by consensus, & thus lacks a general formal definition, but the List of mathematical functions contains functions that are commonly accepted as special. The main aim of this course is the study of basic special functions & proves the properties & relations related to these functions. Furthermore, the simple sets of polynomials are discussed. After the course the students will know & understand the basic concepts of measure theory & the theory of Lebsgue integration. The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs himself, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

1. The Weierstrass gamma function
2. Euler integral representation of gamma function
3. Relations satisfied by gamma function
4. Euler’s constant
5. The order symbols o & O
6. Properties of gamma function
7. Beta function, integral representation of beta function
8. Relation between gamma & beta functions
9. Properties of beta function, Legendre’s duplication formula
10. Gauss’ multiplication theorem
11. Hypergeometric series, the functions F(a,b;c;z) & F(a,b;c;I), integral representation of hypergeometric function,
12. The hypergeometric differential equation, The contiguous relations, Simple transformations,
13. A theorem due to Kummer,
14. Confluent hypergeometric series, Integral representation of confluent hypergeometric function, the confluent hypergeometric,
15. Differential equation, Kummer’s first formula

Recommended Texts

1. Richard, B. (2016). *Special functions & orthogonal polynomials*. Cambridge: Cambridge University Press.
2. Rainville, E. D. (1971). *Special functions* (3rd ed.). New York: The Macmillan Company

Suggested Readings

1. Whittaker, E. T., & Watson, G. N. (1978). *A course in modern analysis*, (2nd ed.). Cambridge: Cambridge University Press.
2. Lebedev, N. N. (1972). *Special functions & their applications* (2nd ed.). New York: Prentice Hall.

MATH-6155 **Theory of Splines-I** **3(3+0)**

This is the first part of the two-course series of Theory of Splines. This course is designed to teach students about basics of scientific computing for solving problems which are generated by data using interpolation & approximation techniques & learn how to match numerical method to mathematical properties. This course gives the students the knowledge of problem classes, basic mathematical & numerical concepts & software for solution of engineering & scientific problems formulated as using data sets. After successful completion, students should be able to design, implement & use interpolations for computer solution of scientific problems involving problems generated by set of data. The material covered provides the studnets with the necessary tools for understanding the many applications of splines in such diverse areas as approximation theory, computer-aided geometric design, curve and surface design and fitting, image processing, numerical solution of differential equations, and increasingly in business and the biosciences.

Contents

1. Basic concepts of Euclidean geometry
2. Scalar & vector functions
3. Barycentric coordinates
4. Convex hull, Matrices of affine maps, Translation, rotation, scaling
5. Reflection & shear, Curve fitting, least squares line fitting
6. Least squares power fit
7. Data linearization method for exponential functions
8. Nonlinear least-squares method for exponential functions
9. Transformations for data linearization
10. linear least squares, Polynomial fitting,
11. Basic concepts of interpolation, Lagrange's method,
12. Rrror terms & error bounds of Lagrange's method
13. Divided differences method,
14. Newton polynomials, error terms & error bounds of Newton polynomials
15. Central difference interpolation formulae
16. Gauss's forward interpolation formula
17. Gauss's backward interpolation formula, Hermite's methods

Recommended Texts

1. David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science + Business Media Inc.
2. John, H. M., & Kurtis, D. F. (1999). *Numerical methods using MATLAB*. New Jersey: Prentice Hall.

Suggested Readings

1. Rao, S. S. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.
2. Sudaran R. K. (1996). *A first course in optimization theory* (3rd ed.). Cambridge: Cambridge University Press.
3. Chang E. K. P., & Zak, S. I. I. (2004). *An introduction to optimization* (3rd ed.). New York: Wiley.

MATH-6156 **Theory of Splines-II** **3(3+0)**

This is the second part of the two-course series of Theory of Splines. The goal of the course is to provide the students with a strong background on numerical approximation strategies & a basic knowledge on the theory of splines that supports numerical algorithms. Interactive graphics techniques for defining & manipulating geometrical shapes used in computer animation, car body design, aircraft design, & architectural design. In this course follow a modular approach & contribute different components to the development of an interactive curve & surface modeling system. Curve Modeling Techniques: Students will implement various curve interpolation & approximation techniques that allow the interactive specification of three-dimensional curves (e.g. Bezier, B-spline, rational curves). Surface modeling techniques: Students will implement various surface interpolation & approximation techniques that allow the interactive specification of three-dimensional surfaces (e.g. Bezier, B-spline, rational surfaces). Simple, 3D Modeling System: Students will integrate the curve & surface modules into a system that allows the user to interactively design & store simple, 3D geometries.

Contents

1. Parametric curves (scalar & vector case), Algebraic form
2. Hermite form, control point form, Bernstein Bezier form
3. Matrix forms of parametric curves
4. Algorithms to compute B.B. form, Convex hull property
5. Affine invariance property, Variation diminishing property
6. Rational quadratic form, Rational cubic form
7. Tensor product surface, B.B. cubic patch
8. Quadratic by cubic B.B. patch, B.B. quartic patch, Splines, Cubic splines
9. End conditions of cubic splines, Clamped conditions
10. Natural conditions, second derivative conditions
11. Periodic conditions, Not a knot conditions
12. General splines, Natural splines, Periodic splines
13. Truncated power function, Representation of spline in terms of truncated power functions
14. Odd degree interpolating splines

Recommended Texts

1. Farin, G. (2002). C*urves & surfaces for computer aided geometric design, a practical guide* (5th ed.). New York: Academic Press.
2. Faux, I. D., & Pratt, M. J. (1979). *Computational geometry for design & manufacture* (1st ed.). New York: Halsted Press.

Suggested Readings

1. Bartle, H. R., & Beatly, C. J. (2006). *An Introduction to spline for use in computer graphics & geometric modeling* (4th ed.). Massachusetts: Morgan Kaufmann.
2. Boor, C. D. (2001). *A practical guide to splines* (Revised ed.). New York: Springer Verlag.

MATH-6157 Methods of Optimization-I 3(3+0)

Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. The objective of this course is to make students acquire a systematic understanding of optimization techniques. The course will start with linear optimization (being the simplest of all optimization techniques) and will discuss in detail the problem formulation and the solution approaches. Then we will cover a class of nonlinear optimization problems where the optimal solution is also globally optimal, i.e. convex nonlinear optimization and its variants. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

1. Introduction to optimization
2. Review of related mathematical concepts
3. Unconstrained optimization
4. Conditions for local minimizers
5. One dimensional search methods
6. Gradient methods
7. Newton's method (analysis & modifications)
8. Conjugate direction methods
9. Quasi Newton method
10. Application to neural network
11. Single Neuron Training
12. Linear integer programming
13. Genetic algorithms
14. Real number genetic algorithm

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory*, (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific

MATH-6158 **Methods of Optimization-II** **3(3+0)**

This is continuation of Methods of Optimization I. Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. Students will learn the foundations of linear programming, properties of optimal solutions and various solution methods for optimizing problems involving a linear objective function and linear constraints. Students will be exposed to geometric, algebraic and computational aspects of linear optimization and its extensions. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

1. Non-linear constrained optimization
2. Problems with equality constraints
3. Problem Formulation
4. Tangent spaces
5. Normal spaces
6. Lagrange condition
7. Second-order conditions
8. Problems with inequality constraints
9. Karush-Kuhn-Tucker Condition
10. Second-order conditions
11. Convex optimization problems
12. Convex functions
13. Algorithms for constrained optimization
14. Lagrangian algorithms

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory,* (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific.

MATH-6159 **Control Theory** **3(3+0)**

This course is an introduction to analysis and design of feedback control systems, including classical control theory in the time and frequency domain. Modeling of physical, biological and information systems using linear and nonlinear differential equations. Stability and performance of interconnected systems, including use of block diagrams, Bode plots, Nyquist criterion, and Lyapunov functions. Robustness and uncertainty management in feedback systems through stochastic and deterministic methods. Introductory random processes, Kalman filtering, and norms of signals and systems. In control system engineering is a subfield of mathematics that deals with the control of continuously operating dynamical system in engineered processes & machines. The objective is to develop a control model for controlling such systems using a control action in an optimum manner without delay or overshoot & ensuring control stability.

Contents

1. System dynamics & differential equations, some system equations
2. System control
3. Mathematical methods & differential equations, The classical & modern control theory
4. Transfer functions & block diagram, Review of Laplace Transforms
5. Applications to differential equations, Transfer functions & Block diagrams
6. State space formations, State space forms, using transfer functions to define state variables, direct solution of the state equation
7. Solutions of the state equation by Laplace transforms, the transformation from companion to the diagonal state form
8. The transform function from the state equation, Transient & steady state response analysis
9. Response of first order system, Response of second order system, Response of higher order systems
10. Steady state error, Feedback control, The concept of stability
11. Routh stability criterion, Introduction to Liapunor's method, Quadratic form
12. Determination of liapunov's function,
13. The Nyquist stability criterion
14. The frequency response

Recommended Texts

1. Burghes, D., & Graham, A. (1980). *Introduction to control theory including optimal control*. New York: Ellis Horwood Ltd.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific.

Suggested Readings

1. Barnett, S., & Camron, R. G. (1985). *Introduction to mathematical control theory* (2nd ed.). Oxford: Oxford V. P.
3. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific.

MATH-6160 Applied Matrix Theory 3(3+0)

Matrix theory is a branch of mathematics which is focused on study of matrices. Initially, it was a sub-branch of linear algebra, but soon it grew to cover subjects related to graph theory, algebra, combinatorics & statistics as well. The aim is to introduce the key mathematical ideas in matrix theory, which are used in modern methods of data analysis, scientific computing, optimization, & merely all quantitative fields of science & engineering. While the choice of topics is motivated by their use in various disciplines, the course will emphasize the theoretical & conceptual underpinnings of this subject, just as in other (applied) mathematics course. The focus of this course is to study the basics of matrices & their applications. Moreover, it concerns with the variational principles, Weyl's inequalities, Gershgorin's theorem & perturbations of the spectrum. The objective is to introduce the key mathematical ideas in matrix theory, which are used in modern methods of data analysis, scientific computing, optimization, & merely all quantitative fields of science & engineering. While the choice of topics is motivated by their use in various disciplines, the course will emphasize the theoretical & conceptual underpinnings of this subject, just as in other (applied) mathematics course.

Contents

1. Eigen values
2. Eigen vectors
3. The Jordan canonical forms
4. Bilinear & quadratic forms
5. Matrix analysis of differential equations
6. Variational principles
7. Perturbation theory
8. The Courant minimax theorem
9. Weyl's inequalities

Recommended Texts

1. Strang, G. (2005). *Linear algebra & its applications*. Cambridge: Academic Press.
2. William, G. (2009). *Linear algebra with applications* (7th ed.). Boston: Allyn & Bacon, Inc.

Suggested Readings

1. Stewart, G. W. (1973). *Introduction to matrix computations*. New York: Academic Press.
2. Franklin, J. N. (2000). *Matrix theory* (1st ed.). New York: Dover Publications.
3. Laub, A. J. (2005). *Matrix analysis for scientsts and engineers*. United States: SIAM.

MSc
MATHEMATICS

MATH-6201 **Real Analysis-I** **3 (3+0)**

This is the first part of a two-semester course. This course covers the fundamentals of mathematical analysis: convergence of sequences & series, continuity, differentiability, Riemann integral, sequences & series of functions, uniformity, & the interchange of limit operations. It shows the utility of abstract concepts & teaches an understanding & construction of proofs. It develops the fundamental ideas of analysis & is aimed at developing the student's ability to describe the real line as a complete, ordered field, to use the definitions of convergence as they apply to sequences, series, & functions, to determine the continuity, differentiability & integrability of functions defined on subsets of the real line, to write solutions to problems & proofs of theorems that meet rigorous st&ards based on content, organization & coherence, argument & support, & style & mechanics, to determine the Riemann integrability of a bounded function & prove a selection of theorems concerning integration, to recognize the difference between pointwise & uniform convergence of a sequence of functions & to illustrate the effect of uniform convergence on the limit function with respect to continuity, differentiability, & integrability.

Contents

1. Number Systems: Ordered fields, rational, real & complex numbers
2. Archimedean property, supremum, infimum & completeness
3. Topology of real numbers: Convergence, completeness
4. Completion of real numbers, intervals
5. Heine Borel theorem
6. Sequences & Series of Real Numbers
7. Limits of sequences, algebra of limits
8. Bolzano Weierstrass theorem
9. Cauchy sequences, liminf, limsup
10. Limits of series, convergences tests
11. Absolute & conditional convergence, power series
12. Continuity: Functions, continuity & compactness
13. Existence of minimizers & maximizers
14. Uniform continuity, continuity & connectedness
15. Intermediate mean value theorem
16. Monotone functions & discontinuities, Differentiation
17. Mean value theorem
18. L'Hopital's Rule, Taylor's theorem

Recommended Texts

1. Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4th ed.). New York: John Wiley & Sons, Inc.
2. Royden, H., & Fitzpatrick, P. (2010). *Real analysis* (4th ed.). London: Pearson.

Suggested Readings

1. Apostal, T. M. (1974). *Mathematical analysis* (2nd ed.). London: Pearson.
2. Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.). New York: McGraw Hill, Inc.

MATH-6202 **Abstract Algebra** **3 (3+0)**

This course aims to provide a first approach to the subject of algebra, which is one of the basic pillars of modern mathematics. The focus of the course will be the study of certain structures called groups, rings, fields & some related structures. Group theory is the study of groups. Groups are sets equipped with an operation (like multiplication, addition, or composition) that satisfies certain basic properties. As the building blocks of abstract algebra, groups are so general & fundamental that they arise in nearly every branch of mathematics & the sciences.This course aims to provide a first approach to the subject of algebra, which is one of the basic pillars of modern mathematics. The focus of the course will be the study of groups, their types & applications. Upon completion of the course, students will be able to demonstrate knowledge and understanding of groups, subgroups, order of an element in finite groups, cosets of a subgroup of a group, normal subgroups, permutation groups, cyclic groups, isomorphism, rings and fields.

Contents

1. Cyclic groups
2. Cosets decomposition of a group
3. Lagrange's theorem & its consequences
4. Conjugacy classes
5. Centralizers & Normalizers
6. Normal Subgroups
7. Homomorphism of groups
8. Cayley's theorem
9. Quotient groups
10. Fundamental theorem of homomorphism
11. Isomorphism theorems
12. Endomorphism & automorphisms of groups
13. Commutator subgroups
14. Permutation groups
15. p-Subgroups, Sylow's theorems
16. Definition & examples of RingsSpecial classes of rings, Fields, Ideals
17. Ring homomorphism

Recommended Texts

1. Gallian, J. A. (2017). *Contemporary abstract algebra* (9th ed.). New York: Brooks/Cole.
2. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.

Suggested Readings

1. Roman, S. (2012). *Fundamentals of group theory* (1st ed.). Basel: Birkhäuser.
2. Rose, H. E. (2006). *A course on finite groups* (1st ed.). London: Springer-Verlag.
3. Fraleigh, J. B. (2003). A *first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.

MATH-6203 **Topology** **3 (3+0)**

Topology studies continuity in its broadest context. We begin by analyzing the notion of continuity familiar from calculus, showing that it depends on being able to measure distance in Euclidean space. This leads to the more general notion of a metric space. A brief investigation of metric spaces shows that they do not provide the most suitable context for studying continuity. A deeper analysis of continuity in metric spaces shows that only the open sets matter, which leads to the notion of a topological spaces. We easily see that this is the right setting for studying continuity. The central concepts of topology, compactness, connectedness & separation axioms are introduced. Applications of topology to number theory, algebraic geometry, algebra & functional analysis are featured. Since many important applications of topology use metric spaces, we investigate topological concepts applied to them & introduce the notion of completeness. In addition, this course provides the basis for studying differential geometry, functional analysis, classical & quantum mechanics, dynamical systems, algebraic & differential topology.

Contents

1. Topological spaces
2. Bases & sub-bases
3. First & second axiom of countability
4. Separability
5. Continuous functions & homeomorphism
6. Finite product space
7. Separation axioms (T_0, T_1, T_2)
8. Techonoff space
9. Regular spaces
10. Completely regular spaces
11. Normal spaces
12. Compact spaces
13. Connected spaces
14. Product spaces
15. Compactness
16. Connectedness

Recommended Texts

1. Lipschutz, S. (1968). *General topology*. Schaum's outline series. New York: McGraw Hill.
2. Sheldon, W. D. (2005). *Topology*. (1st ed.). New York: McGraw Hill.

Suggested Readings

1. Simon, G. F. (1963). *Introduction to topology & modern analysis (*1st ed.). New York: McGraw Hill.
2. Willard, J. (1970). *General topology* (1st ed.). Boston: Addison-Wesley.
3. Munkers, J. R. (2006). *Topology* (2nd ed.). London: Pearson Prentice Hall.

MATH-6204 **Vector & Tensor Analysis** **3 (3+0)**

This course is designed primarily for those students taking courses in mathematics. Vector and tensor algebra have in recent years become basic part of fundamental mathematical background required of those in engineering, sciences and allied disciplines. It is said that vector and tensor analysis is a natural aid in forming mental pictures of physical and geometrical ideas. A most rewarding language and mode of thought for the physical sciences. The focus, therefore, is to impart useful skills on the students in order to enhance their Mathematical ability in applying vector technique to solve problems in applied sciences and to equip them with necessary skill required to cope with higher levels courses in related subjects. Topics to be covered in this course include, basic vector 2 algebra, coordinate bases, gradient, divergence, and curl, Green's, Gauss' and Stokes' theorems. The metric tensor, Christoffel symbols and Riemann curvature tensor. Applications will be drawn from differential geometry, continuum mechanics, electromagnetism, general relativity theory.

Contents

1. Vector Analysis: Gradient
2. Divergence & curl of point functions
3. Expansion formulae
4. Curvilinear coordinates, line, surface & volume integrals
5. Gauss's, Green's & Stoke's theorems
6. Cartesian Tensors: Summation convention
7. Proper & improper transformation
8. Transformation equations
9. Orthogonally conditions
10. Kronecker tensor & Levi-civita tensor
11. Tensors of different ranks
12. Inner & outer products
13. Contraction
14. Quotient theorems
15. Symmetric & anti symmetric tensors
16. Application to Vector Analysis

Recommended Texts

1. Shah, N. A. (2015). *Vector & tenser analysis*. Lahore: Ilmi Ketab Khana.
2. Spiegel, M. R. (2016). *Vector & an introduction to tensor analysis.* New York: McGraw Hill.

Suggested Readings

1. Young, E. C. (1993). *Vector & tensor analysis*. New York: Marcel Dekker, Inc.
2. Brand, L. (2006). *Vector analysis,* New York: Dover Publications.
3. Yousuf, S. M. (1988). *Vector analysis*. Lahore: Ilmi Ketab Khana.

MATH-6205 **Set Theory and ODEs** **3 (3+0)**

The main aim of this course is the study of set theory & the concept of mathematical logic. Everything mathematicians do can be reduced to statements about sets, equality & membership which are basics of set theory. This course introduces these basic concepts. The foundational role of set theory & its mathematical development have raised many philosophical questions that have been debated since its inception in the late nineteenth century. In particular, mathematicians have shown that virtually all mathematical concepts & results can be formalized within the theory of sets. The course aims at familiarizing the students with cardinals, ordinal numbers, relations, functions, Boolean algebra & fundamentals of propositional & predicate logics. This course also introduces the theory, solution, & application of ordinary differential equations. Applications of differential equations in physics, engineering, biology, & economics are presented. The goal of this course is to provide the student with an underst&ing of the solutions & applications of ordinary differential equations.

Contents

1. Set Theory, Equivalent sets, Countable & uncountable sets
2. The concept of cardinal numbers
3. Addition & multiplication of cardinals
4. Cartesian product as sets of functions
5. Addition & multiplication of ordinals
6. Partially ordered sets, axiom of choice
7. The Gamma function
8. The Beta Function
9. Solution in series of Bessel, Legendre & Hyper geometric differential equations
10. Properties & Applications of Bessel function
11. Properties & Applications of Legendre polynomial
12. Generating function & Recurrence relations of Bessel function Legendre polynomials
13. Relations between gamma, beta & hypergeometric function
14. Properties & Applications of hypergeometric function

Recommended Texts

1. Lipschutz, S. (1998). *Schaum's outline of set theory & related topics (*2nd ed.). New York: McGraw Hill.
2. Spiegel, M. R. (2009). *Schaum's outline of advanced mathematics for engineers & scientists.* New York: McGraw Hill.

Suggested Readings

1. Suppes, P. (1972). *Axiomatic set theory.* New York: Dover Publications.
2. Halmos, P. R. (1998). *Naïve set theory*. Berlin: Springer.
3. Temme, N. M. (1995). *Special functions*: *An introduction to the classical functions of mathematical physics* (1st ed.). New York: Wiley-Inter science.

URCI-5109 Introduction to Information and Communication Technologies 3 (2+1)

The course introduces students to information & communication technologies & their current applications in their respective areas. Objectives include basic underst&ing of computer software, hardware, & associated technologies. They can make use of technology to get maximum benefit related to their study domain. Students can learn how the Information & Communications systems can improve their work ability & productivity. How Internet technologies, E-Commerce applications & Mobile Computing can influence the businesses & workplace. At the end of semester, students will get basic underst&ing of Computer Systems, Storage Devices, Operating systems, E-commerce, Data Networks, Databases, & associated technologies. They will also learn Microsoft Office tools that includes Word, Power Point, Excel. They will also learn Open office being used on other operating systems & platforms. Specific software's related to specialization areas are also part of course. Course will also cover Computer Ethics & related Social media norms & cyber laws. The course introduces students to information & communication technologies & their application in the workplace.

Contents

1. Basic Definitions & Concepts
2. Hardware: Computer Systems & Components
3. Storage Devices
4. Number Systems
5. Software: Operating Systems, Programming & Application Software
6. Introduction to Programming
7. Databases & Information Systems Networks
8. Data Communication
9. The Internet, Browsers & Search Engines
10. Email Collaborative Computing & Social Networking
11. E-Commerce
12. IT Security & other issues
13. Use of Microsoft Office tools (Word, Power Point, Excel) or other similar tools depending on the operating system
14. Other IT tools/software specific to field of study of the students if any

Recommended Texts

1. Vermaat. M. E., & Sebok, S. L. (2017). *Discovering computers: digital technology, data & devices*. Russia: Course Technology.
2. Suppes, P. (1972). *Axiomatic set theory*. New York: Dover Publications.

Suggested Readings

1. O'Leary, T., O'Leary, L., & O'Leary, D. (2017). *Computing essentials* (26th ed.). New York: McGraw Hill Higher Education.
2. Fuller, F., & Larson, B. (2015). *Computers*: *underst&ing technology* (5th ed.). Florida: Paradigm,

MATH-6206 **Real Analysis- II** **3 (3+0)**

This course is continuation of Real Analysis I, this course will continue to cover the fundamentals of real analysis, concentrating on the Riemann-Stieltjes integrals, Functions of Bounded Variation, Improper Integrals, & convergence of series. Emphasis would be on proofs of main results. The aim of this course is also to provide an accessible, reasonably paced treatment of the basic concepts & techniques of real analysis for students in these areas. This course provides greatly strengthening student's underst&ing of the results of calculus & the basis for their validitythe uses of deductive reasoning. They will also learn Open office being used on other operating systems & platforms. Specific software's related to specialization areas are also part of course. Course will also cover Computer Ethics & related Social media norms & cyber laws. The course introduces students to information & communication technologies & their application in the workplace.

Content

1. The Riemann Integrals
2. Definition & existence of integrals, properties of integrals
3. Real Valued Functions of Several Variables
4. Continuous real valued functions
5. Partial derivatives & differentials
6. Geometric interpretation of differentiability
7. Chain rule, Taylor's theorem. Maxima & Minima
8. Vector Valued Functions of Several Variables Linear transformations & matrices
9. Continuous & differentiable transformations
10. Chain rule for transformations, Inverse function theorem
11. Implicit function theorem, Jacobians
12. Method of Lagrange multipliers
13. Functions of Bounded Variation
14. Definition & examples, properties of functions of bounded variation
15. Improper Integrals: Types of improper integrals

Recommended Texts

1. Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4th ed.). New York: Wiley.
2. Kaplan, W. (2002). *Advanced calculus* (5th ed.). New York: Addison Wesley.

Suggested Readings

1. Rabenstein, R. L. (1984). *Elements of ordinary differential equations*. Cambridge: Academic Press.
2. Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.). New York: Mac-Graw Hill.
3. Apostal, T. M. (1974). *Mathematical analysis* (2nd ed.). London: Pearson.

MATH-6207 **Linear Algebra** **3(3+0)**

Linear algebra is the study of linear systems of equations, vector spaces, and linear transformations. Solving systems of linear equations is a basic tool of many mathematical procedures used for solving problems in science and engineering. Linear Algebra plays a significant role in many areas of mathematics, statistics, engineering, the natural sciences, and the computer sciences. It provides a foundation of important mathematical ideas that will help students be successful in future coursework. The main objective of this course is to help students to learn in rigorous manner, the tools & methods essential for studying the solution spaces of problems in mathematics and in other fields & develop mathematical skills needed to apply these to the problems arising within their field of study and to various real-world problems. The student will become competent in solving linear equations, performing matrix algebra, calculating determinants, finding eigenvalues & eigenvectors and the student will come to understand a matrix as a linear transformation relative to a basis of a vector space.

Contents

1. Subspaces
2. Bases
3. Dimension of a vector space
4. Quotient space
5. Change of bases
6. Linear Transformation & matrices
7. Inner Product Spaces & Orthognality
8. Orthogonal subspaces
9. Rank & Nullity of linear transformation
10. Eigen values & Eigen vectors
11. Characteristic equation
12. Similar matrices
13. Diagonalization of matrices
14. Orthogonal & Orthonormal set
15. Gramm Schmidt process of orthognalizations
16. Characteristic equation
17. Dual spaces

Recommended Texts

1. Dar, K. H. (2007). *Linear algebra* (1st ed.). Karachi: The Carwan Book House.
2. Kolman, B., & Hill, D. R. (2005). *Introductory linear algebra* (8th ed.). London: Pearson/Prentice Hall.

Suggested Readings

1. Cherney, D., Denton, T., Thomas, R., & Waldron, A. (2013). *Linear algebra* (1st ed.). California: Davis.
2. Anton, H., & Rorres, C. (2014). *Elementary linear algebra: applications version* (11th ed.). New York: John Wiley & Sons.
3. Grossman, S. I. (2004). *Elementary linear algebra* (5th ed.). New York: Cengage Learning.

MATH-6208 Functional Analysis 3(3+0)

This course extends methods of linear algebra & analysis to spaces of functions, in which the interaction between algebra & analysis allows powerful methods to be developed. The course will be mathematically sophisticated & will use ideas both from linear algebra & analysis. This is a basic graduate level course that introduces the student to Functional Analysis & its applications. It starts with a review of the theory of metric spaces, the theory of Banach spaces & proceeds to develop some key theorems of functional analysis. Then continuous to linear operators in Banach & Hilbert spaces & to spectral theory of self-adjoint operators with applications to the theory of boundary value problems, & the theory of linear elliptic partial differential equations. Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn-Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Contents

1. Metric Spaces
2. Convergence
3. Cauchy's sequences & examples
4. Completeness of metric space
5. Completeness proofs
6. Banach Spaces: Normed linear Spaces, Banach Spaces
7. Equivalent norms, Linear operators, Finite dimensional normed spaces
8. Continuous & bounded linear operators
9. Linear functional, Dual spaces
10. Linear operator & functional on finite dimensional Spaces.
11. Inner product Spaces, Hilbert Spaces
12. Conjugate spaces
13. Representation of linear functional on Hilbert space
14. Orthogonal sets, Orthonormal sets & sequences
15. Orthogonal complements & direct sum
16. Reflexive spaces

Recommended Texts

1. Kreyszig, E. (1989). *Introduction to functional analysis with applications*. New York: John Wiley & Sons.
2. Balakrishnan, A. V. (1981). *Applied functional analysis* (2^{nd} ed.). Berlin: Springer-Verlag.

Suggested Readings

1. Dunford, N., & Schwartz, J. T. (1958). *Linear operators. part-1 General theory*. New York: Inter science publishers.
2. Yosida, K. (1981). *Functional analysis* (5^{th} ed.). Berlin: Springer-Verlag.
3. Conway, J. B. (1995). *A course in functional analysis (*2^{nd} ed.). Berlin: Springer-Verlag.

MATH-6209 **Mechanics** **3(3+0)**

The purpose of this course is to provide solid understanding of classical mechanics & enable the students to use this underst&ing while studying courses on quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics & continuum mechanics. The course aims at familiarizing the students with the dynamics of system of particles, kinetic energy, motion of rigid body, Lagrangian & Hamiltonian formulation of mechanics. At the end of this course the students will be able to underst& the fundamental principles of classical mechanics, to master concepts in Lagrangian & Hamiltonian mechanics important to develop solid & systematic problem-solving skills. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn-Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space

Contents

1. Projectile motion with air resistance
2. Applications of the principle of conservation of energy
3. Centre of mass of a system of particles
4. Linear momentum, angular momentum & K.E. with respect to the centre of mass
5. Motion of a rigid body
6. Translation & rotation
7. Linear & angular velocity of a rigid body about a fixed axis
8. Moments & products of inertia
9. Parallel & perpendicular axis theorems
10. General motion of rigid bodies in space
11. Angular momentum & moment of inertia
12. Principal axes & principal moments of inertia
13. Determination of principal axes by diagonalizing the inertia matrix
14. Equi-momental systems
15. Rotating axes theorem
16. Euler's dynamical equations

Recommended Texts

1. Benedetto, D. E. (2011). *Classical mechanics*: *theory & mathematical modeling.* Boston: Birkhauser.
2. Taylor, J. R. (2005). *Classical mechanics*. Colorado: University of Colorado.

Suggested Readings

1. Fowles, G. R., & Cassiday, G. L. (2005). *Analytical mechanics* (7th ed.). Stamford: Thomson Brooks/COLE.
2. Fitzpatrick, R. (2006). *Classical mechanics*. Austin: The University of Texas.
3. Rao, K. S. (2003). *Classical mechanics*. Hyderabad: Universities Press.

MATH-6210 **Complex Analysis** **3(3+0)**

This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis & especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context. When the real numbers are replaced by the complex numbers in the definition of the derivative of a function, the resulting (complex-)differentiable functions turn out to have many remarkable properties not enjoyed by their real analogues. These functions, usually known as holomorphic functions, have numerous applications in areas such as engineering, physics, differential equations and number theory, to name just a few. The focus of this course is on the study of holomorphic functions and their most important basic properties.

Contents

1. Introduction: The algebra of complex numbers
2. Geometric representation of complex numbers, Polar form of complex numbers
3. Powers & roots of complex numbers
4. Functions of Complex Variables, Limit, Continuity, & Differentiable functions
5. The Cauchy-Riemann equations, Analytic functions
6. Entire functions, Harmonic functions
7. Elementary functions: The exponential, Trigonometric, Hyperbolic, Logarithmic & Inverse elementary functions
8. Complex Integrals: Contours & contour integrals
9. Antiderivatives, Independence of path
10. Cauchy-Goursat theorem, Cauchy integral formula
11. Lioville's theorem, Morerea's theorem
12. Maximum Modulus Principle
13. Series: Power series, Radius of convergence & analyticity
14. Taylor's & Laurent's series
15. Integration & differentiation of power series
16. Isolated singular points
17. Cauchy's residue theorem with applications
18. Types of singularities, Calculus of residues, Zeros & Poles, Mobius transforms
19. Conformal mappings & transformations

Recommended Texts

1. Churchill, R. V., & Brown, J. W. (1989). *Complex variables & applications* (5th ed.). New York: McGraw Hill.
2. Mathews, J. H., & Howell, R. W. (2006). *Complex analysis for mathematics & engineering*. Sudbury: Jones & Bartlett.

Suggested Readings

1. Lang, S. (1999). *Complex analysis*. Berlin: Springer-Verlag.
2. Remmert, R. (1991). *Theory of complex functions*. Berlin: Springer-Verlag.
3. Rudin, W. (1987). *Real & complex analysis*. New York: McGraw-Hill.

MATH-6211 Computer Programming with C++ 3(2+1)

The basic objective for this course to provide complete guidance for the students to learn object-oriented programming technique. The idea is to focus on practical aspect of programming in Mathematics. Programming Languages plays an important role in Mathematics. A number of computer software available to deal with mathematical computing & simulation. After this course students will be able to develop computer programs in this software according to their requirements in mathematical computing. This course introduces the student to programming through a study of the concepts of program specification and design, algorithm development, and coding and testing using a modern software development environment. Students learn how to write programs in an object-oriented high-level programming language. Topics covered include fundamentals of algorithms, flowcharts, problem solving, programming concepts, classes and methods, control structures, arrays, and strings. Throughout the semester, problem solving skills will be stressed and applied to solving computing problems. Weekly laboratory experiments will provide hands-on experience in topics covered in this course.

Contents

1. Introduction to operating systems
2. C language
3. Building blocks
4. Variables
5. Input/output
6. loops (FOR, WHILE, DO)
7. Decisions (IF, IF ELSE, ELSE IF) construct switch statement
8. Conditional statement
9. Function hat returns a value using argument to pass data to another function
10. External variable
11. Predefined variable
12. Arrays
13. Strings
14. Pointers
15. Structure
16. Files & introduction to C++

Recommended Texts

1. Aho, A.V., & Ulman, J. D. (1995). *Foundation of computer science* (1st ed.). New York: Computer Science Press, W. H. Freeman.
2. Mustafa, T., Tariq, M., & Imran, S. (2017). *Object oriented programming using C++* (2nd ed.). Pakistan: IT series.

Suggested Readings

1. Hein, J. L. (1996). *Theory of computation: an introduction* (1st ed.). Boston: Jones & Bartlett.
2. Deitel, P. & Deitel, H. (2013). *C for programmers with an introduction to C++*. New Jersey: Prentice Hall.
3. Prata, S. (2013). *C primer plus* (6th ed.). Boston: Addison-Wesley Professional.

URCC-5110 Citizenship Education and Community Engagement 3 (1+2)

In recent years, community engagement has become a central dimension of governance as well as policy development & service delivery. However, efforts to directly involve citizens in policy processes have been bedeviled by crude underst&ings of the issues involved, & by poor selection of techniques for engaging citizens. This course will provide a critical interrogation of the central conceptual issues as well as an examination of how to design a program of effective community engagement. This course begins by asking: Why involve citizens in planning & policymaking? This leads to an examination of the politics of planning, conceptualizations of "community" &, to the tension between local & professional knowledge in policy making. This course will also analyze different types of citizen engagement & examine how to design a program of public participation for policy making. Approaches to evaluating community engagement programs will also be a component of the course. Moreover, in order to secure the future of a society, citizens must train younger generations in civic engagement & participation. Citizenship education is education that provides the background knowledge necessary to create an ongoing stream of new citizens participating & engaging with the creation of a civilized society.

Contents

1. Introduction to Citizenship Education & Community Engagement: Orientation
2. Introduction to Active Citizenship: Overview of the ideas, Concepts, Philosophy & Skills
3. Identity, Culture & Social Harmony: Concepts & Development of Identity
4. Components of Culture & Social Harmony, Cultural & Religious Diversity
5. Multi-cultural society & inter-cultural dialogue: bridging the differences, promoting harmony
6. Significance of diversity & its impact, Importance & domains of inter-cultural harmony
7. Active Citizen: Locally active, Globally connected
8. Importance of active citizenship at national & global level, underst&ing community
9. Identification of resources (human, natural & others), Human rights, Universalism vs relativism
10. Constitutionalism & citizens' responsibilities: Introduction to human rights
11. Human rights in constitution of Pakistan, Public duties & responsibilities
12. Social Issues in Pakistan: Introduction to the concept of social problem, Causes & solutions
13. Social Issues in Pakistan (Poverty, Equal & Equitable access of resources, unemployment)
14. Social Issues in Pakistan (Agricultural problems, terrorism & militancy, governance issues)

Recommended Books

1. Kennedy, J. K., & Brunold, A. (2016). *Regional context & citizenship education in Asia & Europe*. New York: Routledge Falmer.
2. Macionis, J. J., & Gerber, M. L. (2010). *Sociology*. New York: Pearson Education.

Suggested Books

1. Council, B. (2017). *Active citizen's social action projects guide.* Scotland: British Council.
2. Larsen, K. A. (2013). *Participation in community work: international perspectives.* Vishanthic Sewpaul, Grete Oline Hole.

MATH-6212 **Differential Geometry** **3(3+0)**

Differential geometry is the study of geometric properties of curves, surfaces, & their higher dimensional analogues using the methods of calculus. It has a long & rich history, &, in addition to its intrinsic mathematical value & important connections with various other branches of mathematics, it has many applications in various physical sciences, e.g., solid mechanics, computer tomography, or general relativity. Differential geometry is a vast subject. This course covers many of the basic concepts of differential geometry in the simpler context of curves & surfaces in ordinary 3-dimensional Euclidean space. The aim is to build both a solid mathematical underst&ing of the fundamental notions of differential geometry & enough visual & geometric intuition of the subject. This course is of interest to students from a variety of math, science & engineering backgrounds, & that after completing this course, the students will ready to study more advanced topics such as global properties of curves & surfaces, geometry of abstract manifolds, tensor analysis, & general relativity.

Contents

1. Space Curves
2. Arc length
3. Tangent
4. Normal & binormal
5. Curvature & torsion of a curve
6. Tangent planes
7. The Frenet-Serret apparatus
8. Fundamental existence theorem of plane curves
9. Four vertex theorem, Isoperimetric inequality
10. Surfaces
11. First fundamental form
12. Isometry & conformal mappings
13. Curves on Surfaces
14. Surface Area
15. Second fundamental form
16. Normal & Principle curvatures
17. Gaussian & Mean curvatures
18. Geodesics

Recommended Texts

1. Pressley, A. (2001). *Elementary differential geometry*. Berlin: Springer-Verlag.
2. Somasundaran, D. (2005). *Differential geometry.* New Delhi: Narosa Publishing House.

Suggested Readings

3. Weatherburn, C. E. (1955). *Differential geometry*. Cambridge: Cambridge University Press.
4. Millman, R. S., & Parker, G. D. (1977). *Elements of differential geometry*. New Jersey: Prentice Hall.
5. Wilmore, T. J. (1959). *An introduction to differential geometry*. Oxford: Oxford University Press.

MATH-6213 Partial Differential Equations 3(3+0)

Partial Differential Equations (PDEs) are in the heart of applied mathematics & many other scientific disciplines. The beginning weeks of the course aim to develop enough familiarity & experience with the basic phenomena, approaches, & methods in solving initial/boundary value problems in the contexts of the classical prototype linear PDEs of constant coefficients: the Laplace equation, the wave equation & the heat equation. A variety of tools & methods, such as Fourier series/eigenfunction expansions, Fourier transforms, energy methods, & maximum principles will be introduced. This course is of interest to students from a variety of math, science & engineering backgrounds, & that after completing this course, the students will ready to study more advanced topics such as global properties of curves & surfaces, geometry of abstract manifolds, tensor analysis, & general relativity.

Contents

1. First order PDEs: Introduction, Formation of PDEs, Solutions of PDEs of first order
2. The Cauchy's problem for quasi linear first order PDEs, First order nonlinear equations
3. Special types of first order equations Second order PDEs
4. Basic concepts & definitions, Mathematical problems
5. Linear operator
6. Superposition, Mathematical models
7. Conduction of heat solids, Canonical forms & variable
8. PDEs of second order in two independent variables with constant & variable coefficients
9. Cauchy's problem for second order PDEs in two independent variables
10. Methods of separation of variables, Solutions of elliptic PDEs in Cartesian & cylindrical coordinates
11. Solutions of Parabolic & hyperbolic PDEs in Cartesian & cylindrical coordinates
12. Laplace transform: Introduction & properties of Laplace transform
13. Transforms of elementary functions
14. Periodic functions, error function & Dirac delta function
15. Inverse Laplace transform, Convolution Theorem
16. Solution of PDEs by Laplace transform
17. Diffusion & wave equations, Fourier transforms

Recommended Texts

1. Zill, D. G., & Cullen M. R. (2008). *Differential equations with boundary value problems.* New York: Brooks Cole.
2. Polking, J., & Boggess A. (2005). *Differential equations with boundary value problems* (2^{nd} ed.). London: Pearson.

Suggested Readings

1. Humi., M., & Miller, W. B. (1991). *Boundary value problems & partial differential equations*. Boston: PWS- KENT Publishing Company.
2. Myint, U.T. (1987). *Partial differential equations for scientists & engineers* (3^{rd} ed.). Amsterdam: North Holland.

MATH-6214 **Numerical Analysis-I** **3(3+0)**

To explore complex systems, physicists, engineers, financiers & mathematicians require computational methods since mathematical models are only rarely solvable algebraically. Numerical methods, based upon sound computational mathematics, are the basic algorithms underpinning computer predictions in modern systems science. The course will cover the classical fundamental topics in numerical methods such as, approximation, numerical linear algebra, solution of nonlinear algebraic systems, matrix decomposition & unstable systems. The viewpoint will be modern, with connections made between each topic & a variety of applications. This course is of interest to students from a variety of math, science & engineering backgrounds, & that after completing this course, the students will ready to study more advanced topics such as global properties of curves & surfaces, geometry of abstract manifolds, tensor analysis, & general relativity.

Contents

1. Computer arithmetic, approximations & errors
2. Linear & non-linear equations
3. Methods for solution of linear equations
4. Gaussian elimination method, Gauss-Jordan method
5. Crout's method, Cholesky's method
6. Doolittle's method
7. LU-factorization & Matrix inversion
8. Iterative methods: Jacobi, Gauss-Seidel
9. error analysis for iterative methods
10. Methods for the solution of nonlinear equations
11. Bisection method
12. Regula-falsi method
13. Fixed point iteration method
14. Newton-Raphson method, Secant method
15. Interpolation: polynomial approximation
16. Forward, backward & centered difference formulae
17. Lagrange interpolation, Newton's divided difference formula
18. Interpolation with a cubic spline
19. Hermite interpolation

Recommended Texts

1. Burden, R. L., & Faires, J. D. (2010). *Numerical analysis* (9th ed.). New York: Brooks /Cole.
2. Geddes, M., & Labahn. (2001). *Maple 7 programming guide*. Waterloo: Maple Inc.

Suggested Readings

1. Jaan, K. (2013). *Numerical methods in engineering with python 3*. Cambridge: Cambridge University Press.
2. Gerald, C. (1984). *Applied numerical analysis*. Boston: Addison-Wesley.
3. Balfour A., & Beveridge W. T. (1977). *Basic numerical analysis with FORTARAN*. Portsmouth: Heinmann Educational Books Ltd.

MATH-6215 Probability Theory 3(3+0)

This course is the introduction to the Probability theory, which is the branch of mathematics that deals with modelling uncertainty. It is important because of its direct application in areas such as genetics, finance and telecommunications. It also forms the fundamental basis for many other areas in the mathematical sciences including statistics, modern optimisation methods and risk modelling. A prime objective of the course is to introduce the students to the fundamentals of probability theory & present techniques & basic results of the theory & illustrate these concepts with applications. This course will also present the basic principles of random variables & random processes needed in 24 applications. After the course, the students should be able to understand axioms of Probability theory and have working knowledge with probability calculations behind hierarchical model building; be aware of different types of convergence and to be able to visualize basic concepts of probability theory using Matlab.

Contents

1. Finite probability spaces
2. Basic concept
3. probability & related frequency
4. Combination of events, examples
5. Independence, random variables
6. Expected value
7. Standard deviation
8. Chebyshev's inequality
9. Independence of r&om variables
10. Multiplicatively of the expected value
11. Additivity of the variance
12. Discrete probability distribution
13. Probability as a continuous set function
14. Sigma-algebras, examples
15. Continuous r&om variables
16. Expectation & variance
17. Normal r&om variables & continuous probability distribution
18. Applications: De Moivre-Laplace limit theorem
19. Weak & strong law of large numbers
20. The central limit theorem
21. Markov chains & continuous Markov process

Recommended Texts

1. Capinski, M., & Kopp, E. (1998). *Measure, integral & probability.* Berlin: Springer-Verlag.
2. Dudley, R. M. (2004). *Real analysis & probability.* Cambridge: Cambridge University Press.

Suggested Readings

1. Resnick, S. I. (1999). *A probability path*. Basel: Birkhauser.
2. Ross, S. (1998). *A first course in probability theory* (5th ed.). New Jersey: Prentice Hall.
3. Ash, R. B. (2008). *Basic probability theory*. New York: Dover Books.

MATH-6216 Integral Equations 3(3+0)

Many physical problems that are usually solved by differential equation methods can be solved more effectively by integral equation methods. This course will help students gain insight into the application of advanced mathematics & guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems. In addition, a large class of initial & boundary value problems, associated with the differential equations, can be reduced to the integral equations, whence enjoy the advantage of the above integral presentations. This course has many applications in many sciences. This course emphasizes concepts and techniques for solving integral equations from an applied mathematics perspective. Material is selected from the following topics: Volterra and Fredholm equations, Fredholm theory, the Hilbert-Schmidt theorem; Wiener-Hopf Method; Wiener-Hopf Method and partial differential equations; the Hilbert Problem and singular integral equations of Cauchy type; inverse scattering transform; and group theory. Examples are taken from fluid and solid mechanics, acoustics, quantum mechanics, and other applications.

Contents

1. Introduction to Integral equations
2. Physical applications of Integral equation
3. Linear integral equations of the first kind
4. Linear integral equations of the second kind
5. Relationship between differential equation & Volterra integral equation
6. Neumann series
7. Fredholm Integral equation of the second kind with separable Kernels
8. Eigen values & eigenvectors
9. Iterated functions
10. Quadrature methods
11. Least square methods
12. Homogeneous integral equations of the second kind
13. Fredholm integral equations of the first kind
14. Fredholm integral equations of the second kind

Recommended Texts

1. Jerri, A. J. (2007). *Introduction to integral equations with applications* (2nd ed.). New York: Marcel Dekker Inc
2. Li, X. (2013). *Integral equations, boundary value problems and related problems* (1st ed.). Singapore: World Scientific Publishing Company.

Suggested Readings

1. Lovitt, W. V. (2005). *Linear integral equations.* New York: Dover Publications.
2. Wazwaz, A.M. (2011). *Linear & nonlinear integral equations methods & application.* Berlin: Springer.
3. Christian, C., Dale, D., & Hamill, W. (2014). *Boundary integral equation methods & numerical solutions* (1sted.). Berlin: Springer.

MATH-6217 **Numerical Analysis-II** **3(3+0)**

To explore complex systems, physicists, engineers, financiers & mathematicians require computational methods since mathematical models are only rarely solvable algebraically. Numerical methods, based upon sound computational mathematics, are the basic algorithms underpinning computer predictions in modern systems science. The course will cover the classical fundamental topics in numerical methods such as, numerical integration, solution of ordinary & partial differential equations. Linear operators, multi-step methods & solution of difference equations of different types. The viewpoint will be modern, with connections made between each topic & a variety of applications. Despite the many solution techniques presented in elementary calculus and differential equations courses, mathematical models used in applications often do not have the simple forms required for using these methods. Hence, a quantitative understanding of the models requires the use of numerical approximation schemes. This course provides the mathematical background for understanding how such schemes are derived and when they are likely to work.

Contents

1. Numerical Integration: Trapezoidal
2. Numerical Integration: Simpson 1/3 rule
3. Numerical Integration: Simpson's 3/8 rule
4. Quadrature formulas for evaluating integrals with error estimates,
5. Newton-Cotes Formulae: Open & closed Newton-Cotes formulae
6. Difference & Differential Equation: Formulation of difference equations
7. Solution of linear (homogeneous & inhomogeneous) difference equations with constant coefficients
8. The Euler & modified Euler method
9. Runge-Kutta methods & multistep methods
10. Predictor-corrector type methods for solving initial value problems along with convergence & instability criteria
11. Finite difference
12. Collocation & variational method for boundary value problems
13. Maple Programming to implement above mentioned topics

Recommended Texts

1. Burden, R. L., & Faires, J. D. (2010). *Numerical analysis* (9th ed.). New York: Brooks /Cole.
2. Geddes, M. & Labahn. (2009). *Maple 7 programming guide*. Waterloo: Maple Inc.

Suggested Readings

1. Jaan, K. (2013). *Numerical methods in engineering with python 3*. Cambridge: Cambridge University Press.
2. Gerald, C. (1984). *Applied numerical analysis*. New York: Addison-Wesley.
3. Balfour, A., & Beveridge, W. T. (1977). *Basic numerical analysis with FORTARAN*. Portsmouth: Heinmann Educational Books Ltd.

MATH-6218 History of Mathematics 3(3+0)

This course is designed to provide the historical background to some of the mathematics familiar to students. This course is a survey of the historical development of mathematics. The emphasis will be on mathematical concepts, problem solving, and pedagogy from a historical perspective. In this course, we will explore some major themes in mathematics calculation, number, geometry, algebra, infinity, formalisms & their historical developments in various civilizations. We will see how the earlier civilizations influenced or failed to influence later ones & how the conceps evolved in these various civilizations. The aims of teaching & learning mathematics are to encourage & enable students to understand & be able to use the language, symbols & notation of mathematics, develop mathematical curiosity & use inductive & deductive reasoning when solving problems. Students will demonstrate their knowledge of basic historical facts; they will demonstrate understanding of the development of mathematics and mathematical thought.

Contents

1. History of Numerations
2. Egyptian
3. Babylonian
4. Hindu contributions
5. Arabic contributions
6. Algebra: Including the contributions of Al-Khwarzmi
7. Algebra: Including the contributions of Ibn Kura
8. History of Geometry
9. History of Euclud’s elements
10. History of Analysis
11. The Calculus: Newton
12. The Calculus: Leibniz
13. The Calculus: Gauss
14. The contributions of Bernoulli brothers
15. The Twentieth Century Mathematics

Recommended Texts

1. Boyer, B., & Mersbach, U. V. (1989). *The history of mathematics* (2nd ed.). London: Jossey-Bass.
2. Berlinghoff, William, P., & Fernando, Q. G. (2004). *Math through the ages: A gentle history for teachers & others,* (Expanded ed.). Washigton: Oxton House & MAA.

Suggested Readings

1. Burton, D. M. (2011). *The history of mathematics: an introduction* (7th ed.). New York: McGraw-Hill.
2. Katz, V. J. (2009). *A history of mathematics, an introduction* (3rd ed.). Boston: Addison-Wesley.
3. Dunham, W. (1990). *Journey through genius: The great theorems of mathematics*. London: Penguin Books.

MATH-6219 Special Functions 3(3+0)

Special functions are particular mathematical functions that have more or less established names & notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.The term is defined by consensus, & thus lacks a general formal definition, but the List of mathematical functions contains functions that are commonly accepted as special.The main aim of this course is the study of basic special functions & proves the properties & relations related to these functions. Furthermore, the simple sets of polynomials are discussed. This course will introduce participants to the concepts, methods, and tools for special functions. This course will cover the full spectrum of the development of the theory of special functions as the solution of differential equations along with their numerical aspect. We will also discuss how one can explore this theory in other areas of mathematics, namely, Fourier analysis, complex analysis, and fluid dynamics, etc.

Contents

1. The Weierstrass gamma function
2. Euler integral representation of gamma function
3. Relations satisfied by gamma function
4. Euler’s constant, The order symbols o & O
5. Properties of gamma function
6. Beta function, integral representation of beta function
7. Relation between gamma & beta functions
8. Properties of beta function
9. Legendre’s duplication formula
10. Gauss’ multiplication theorem
11. Hypergeometric series, the functions F(a,b;c;z) & F(a,b;c;I), integral representation of hypergeometric function,
12. The hypergeometric differential equation, The contiguous relations, Simple transformations,
13. A theorem due to Kummer,
14. Confluent hypergeometric series, Integral representation of confluent hypergeometric function, the confluent hypergeometric,
15. Differential equation, Kummer’s first formula

Recommended Texts

1. Richard, B. (2016). *Special functions & orthogonal polynomials.* Cambridge: United Kingdom Cambridge University Press.
2. Rainville, E. D. (1971). *Special function* (3rd ed.). New York: The Macmillan Company.

Suggested Readings

1. Whittaker, E. T., & Watson, G. N. (1978). *A Course in modern analysis* (2nd ed.). Cambridge: University Press.
2. Lebedev, N. N. (1972). *Special functions & their applications* (2nd ed.). New York: Prentice Hall.

MATH-6220 Algorithms and Computer Programming 3(2+1)

This course is designed to teach the students about numerical methods & their theoretical bases. The course aims at inculcating in the students the skill to apply various techniques in numerical analysis, underst& & do calculations about errors that can occur in numerical methods & underst& & be able to use the basics of matrix analysis. This is a lecture & laboratory course offered to introduce computer science & programming. Topics include information & verification of numerical & approximated methods given in numerical analysis, solutions of ODEs & PDEs, Finite difference methods & many more. It is optimal to verifying numerical methods by using computer programming (Matlab, Mathematica, Maple, C++, Fortran). Computer programming courses focus on helping students develop an understanding of computer networks, operating systems, algorithms, database systems and web design. Students in computer programming courses will become familiar with programming languages such as HTML and CSS.

Contents

1. Iterative Method
2. Bisection method
3. Regula Falsi Method
4. Newton-Raphson method for solving non-linear equations
5. Gaussian elimination with different pivoting strategies
6. Newton's interpolation
7. Lagranges interpolation
8. Jacobi & Gauss-Seidal Iterative methods for systems of simultaneous linear equations,
9. Trapezoidal rule, Simpson's rule
10. Gaussian method of numerical integration
11. Modified Euler's methods
12. Improved Euler's methods
13. Predictor corrector methods for finding the numerical solution of IVP's involving ODE'S
14. Predictor corrector methods for finding the numerical solution of BVP's involving ODE'S
15. Practical examination will be of two hours duration in which one or more computational projects will be examined.

Recommended Texts

1. Abell, M. L., & Braselton, J. P. (1992). *Mathematica handbook*. Amsterdam: Elsevier.
2. Akai, T. J. (1994). *Applied numerical methods.* New York: Willey & Sons Inc.

Suggested Readings

1. Mathews, J. (1987). *Numerical methods for computer science, engineering & mathematics*. New Jersey: Prentice Hall.
2. Jaan, K. (2013). *Numerical methods in engineering with python 3*. Cambridge: Cambridge University Press.
3. Gerald, C. (1984). *Applied numerical analysis*. Boston: Addison-Wesley.
4. Balfour, A., & Beveridge W. T. (1977). *Basic numerical analysis with FORTARAN*. Heinmann Educational Books Ltd.

MATH-6221 Advanced Programming for Scientific Computing 3(2+1)

The basic objective for this course to provide complete guidance for the students to learn object-oriented programming technique. This course covers the techniques required to develop C++ programs that solve mathematical & scientific problems. As well as covering the rudiments of C++ (which will be taught with no assumed prior knowledge) the course will also outline the basic techniques used in scientific programming, such as discretisation of equations, numerical error & code validation. The material is examined primarily through two programming projects, chosen from a list of mathematical topics, which investigate particular algorithms or techniques in more depth. The projects will be assessed by a written report & a demonstration/oral description of the code. Much of this course is taught in practical computer labs, which limits the number of places available. The idea is to focus on practical aspect of programming in Mathematics. Programming Languages plays an important role in Mathematics.

Contents

1. Introduction to Object Oriented Programming (OOP)
2. Using OOP Features
3. Overloading Overriding Inheritance Polymorphism
4. Decisions (IF, IF ELSE, ELSE IF) construct switch statement
5. Conditional statement
6. Function hat returns a value using argument to pass data to another function
7. External variable
8. Arrays & strings
9. Writing Programs for Numerical Methods Mixed Programming with FORTRAN/C++/Matlab

Recommended Texts

1. Deitel, H. M., & Deitel, P. J. (2007). *C++ how to program* (5th ed.). Sudbury: Deitel & Associates.
2. Chapenman, S. J. (2003). *FORTRAN 90/95 for scientists & engineers* (2nd ed.). New York: McGraw Hill.

Suggested Readings

1. Press, W. H. (1989). *Numerical recipes in FORTRAN (*2nd ed.). Cambridge: Cambridge University Press.
2. Hein, J. L. (1996). *Theory of computation: an introduction* (1st ed.). Boston: Jones & Bartlett.
3. Paul, D., & Harvey, D. (2013). *C for programmers with an introduction to C++*. New Jersey: Prentice Hall.

MATH-6222 **Analytical Number Theory** **3(3+0)**

In mathematics, analytic number theory is a branch of number theory that uses methods from mathematical analysis to solve problem about the integers. Analytic number theory aims to study number theory by using analytic tools (inequalities, limits, calculus, etc). In this course we will mainly focus on studying the distribution of prime numbers by using analysis. Perhaps it is surprising that such a link even exists. We will try & give precise answers to questions such as Roughly how big is the nth prime? Approximately how many primes are less than a given number? How fast does the sequence of primes diverge to infinity? Is there always a prime between n & 2n? A big result to be proved in the course is Dirichlet's theorem on primes in arithmetic progressions. This tells us that for coprime a,b the sequence an+b contains infinitely many primes. The general proof of this result features a unique blend of algebra & analysis & was the cornerstone of 19th century number theory

Contents

1. Divisibility
2. Euclid’s theorem
3. Congruences
4. Elementary properties
5. Residue classes
6. Euler’s function
7. Linear congruence
8. congruence of higher degree
9. Congruences with prime moduli
10. The theorems of Fermat
11. Euler & Wilson
12. Primitive roots & indices
13. Integers belonging to a given exponent
14. Composite moduli Indices
15. Quadratic Residues, Composite moduli
16. Legendre symbol
17. Law of quadratic reciprocity
18. The Jacobi symbol
19. Number-Theoretic Functions,
20. Mobius function,
21. The function [x],
22. Diophantine Equations, Equations & Fermat’s conjecture for n = 2, n = 4

Recommended Texts

1. Griffin, H. (1970). *Elementary theory of numbers.* New York: McGraw Hill.
2. William, J. L. (2002). *Topics in number theory, volumes I & II.* New York: Dover Publications.

Suggested Readings

1. Leveque, W. J. (2002). Topics *in number theory, Vol. I*. Boston: Addison-Wesley.
2. Burton, D. M. (2007). *Elementary number theory*. New York: McGraw-Hill.

MATH-6223 **Algebraic Number Theory** **3(3+0)**

The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, & a key objective of the course is to introduce the class group which measures the failure of this property. Algebraic number theory is a branch of number theory that uses the techniques of abstract algebra to study the integers, rational numbers, & their generalizations. One of the main goals of algebraic number theory is to understand how the arithmetic of number fields allows us to classify solutions to certain Diophantine equations. For example, we saw how understanding the arithmetic of cyclotomic fields could help us solve certain cases of Fermat's Last Theorem.

Contents

1. Review of polynomials
2. Irreducible polynomials
3. Algebraic numbers & integers
4. Units & Primes in R [v] ideals
5. Arithmetic of ideals congruencies
6. The norm of an ideal
7. Prime ideals
8. Units of algebraic number field
9. Equivalence
10. Class number
11. Cyclotomic field K_p
12. Fermat's equation
13. Kummer's theorem
14. The equation x2 + 2 =y^3
15. pure cubic fields
16. Distribution of primes
17. Riemann's zeta function

Recommended Texts

1. Leveque, W. J. (2002). *Topics in number theory, volumes I & II.* New York: Dover Publishers.
2. Stewart, I. N., & Tall, D.O. (2002). *Algebraic number theory* (2^{nd}ed.*)*. Florida: Chapman & Hall/CRC Press.

Suggested Readings

1. Leveque, W. J. (1956). *Topics in number theory, Vol. II.* Boston: Addison-Wesley.
2. Marcus, D. (1977). *Number fields*. Berlin: Springer-Verlag.

MATH-6224 **Advance Group Theory-I** **3(3+0)**

This is the first part of the two advance course series of Group Theory. This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outst&ing beauty. It takes just three simple axioms to define a group, & it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts & results of Group Theory.

Contents

1. Group of automorphisms
2. Inner automorphisms, definition & related results
3. Characteristic & fully invariant subgroups
4. Symmetric Groups
5. Cyclic permutations, even & odd permutations
6. The alternating groups
7. Conjugacy classes of symmetric & alternating groups
8. Generators of symmetric & alternating groups
9. Simple groups, simplicity of symmetric & alternating groups
10. Group Action on sets or G-sets, Orbits & stabilizer subgroups
11. Finite direct products
12. Finitely generated abelian groups
13. P-groups, Sylow's Theorems
14. Application of Sylow's Theorems
15. Linear Groups, Types of Linear Groups, Classical Groups

Recommended Texts

1. Fraleigh, J. B. (2003). *A first course in abstract algebra (*7th ed.). Boston: Addison Wesley Publishing Company.
2. Shah, S. K., & Shankar, A. G. (2013). *Group theory*. London: Dorling Kindersley.

Suggested Readings

1. Bhattacharya, P. B., & Jain, S. K., Nagpaul S. R. (1994). *Basic abstract algebra (*2nd ed.). Cambridge: Cambridge University Press.
2. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: McGraw-Hill.
3. Rotman, J. J. (1999). *An introduction to the theory of groups* (4thed.). Berlin: Springer.

MATH-6225 **Advance Group Theory-II** **3(3+0)**

This course is the continuation of the course "Advanced Group Theory-1". This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. This course covers the advanced topics in group theory such as solvable groups, Upper & Lower Central seriesnilpotent groups & free groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outst&ing beauty. It takes just three simple axioms to define a group, & it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts & results of Group Theory.

1. Series in groups
2. Normal series
3. Normal series & its refinement
4. Composition series
5. Equivalent composition series
6. Jordan Holder Theorem
7. Solvable groups, definition, examples & related results
8. Upper & Lower Central series
9. Nilpotent groups
10. Characterization of finite nilpotent groups
11. The Frattini subgroups, definition, examples & related results
12. Free groups, definition, examples & related results
13. Free Product, definition, examples & related results
14. Group algebras
15. Representation modules

Recommended Texts

1. Fraleigh, J. B. (2003). *A first course in abstract algebra* (7th ed.). Boston: Addison Wesley Publishing Company.
2. Shah, S. K., & Shankar, A. G. (2013). *Group theory*. London: Dorling Kindersley.

Suggested Readings

1. Bhattacharya, P. B., & Jain, S. K., Nagpaul S. R. (1994). *Basic abstract algebra (*2nd ed.). Cambridge: Cambridge University Press.
2. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract algebra*. New York: McGraw-Hill.
3. Rotman, J. J. (1999). *An introduction to the theory of groups* (4thed.). Berlin: Springer.

MATH-6226 Algebraic Topology-I 3(3+0)

The course gives an introduction to algebraic topology, with emphasis on the fundamental group and the singular homology groups of topological spaces. This course aims to understand some fundamental ideas in algebraic topology; to apply discrete, algebraic methods to solve topological problems; to develop some intuition for how algebraic topology relates to concrete topological problems. The primary aim of this course is to explore properties of topological spaces. We shall consider in detail examples such as surfaces. To distinguish topological spaces, we need to define topological invariants, such as the "fundamental group" or the "homology" of a space". To enable us to do this, knowledge of basic group theory & topology is essential. Some background in real analysis would also be helpful. After completing the course students can work with cell complexes and the basic notions of homotopy theory, know the construction of the fundamental group of a topological space, can use van Kampen´s theorem to calculate this group for cell complexes and know the connection between covering spaces and the fundamental group.

Contents

1. Affin spaces
2. Singular theory
3. Chain complexes
4. Homotopy invariance of homology
5. Relation between n, & H,relative homology
6. The exact homology sequence
7. Nilpotent groups
8. Homotopy theory
9. Homotopy theory of path & maps
10. Fundamental group of circle
11. Covering spaces
12. Lifting criterion
13. Loop spaces
14. Higher homotopy group.

Recommended Texts

1. Kosniowski, C. A. (1980). *First course in algebraict topology*. Cambridge: Cambridge University Press.
2. Greenberg, M. J., & Harper, J. R. (1967). *Algebraic topology, a first course.* Colorado: Westview Press.

Suggested Readings

1. Croom, F. H. (1978). *Basic concepts of algebraic theory*. Berlin: Springer.
2. Adhikari, M. R. (2016). *Basic algebraic topology & its applications* (1st ed.). Berlin: Springer.
3. Hatcher, A. (2001). *Algebraic topology*. Cambridge: Cambridge University Press.

MATH-6227 **Algebraic Topology-II** **3(3+0)**

This course is a continuation of Algebraic Topology-I. In this course, the objective is the study of knots, links, surfaces & higher dimensional analogs called manifolds with the underst&ing that continuous deformations do not change objects. So, a doughnut (torus) & a coffee mug are essentially the same (homeomorphic) in this course. For example, how does a creature living on a sphere tell that she is not on the plane, on the torus, or perhaps a two holed torus? Can one turn a sphere inside out without creasing it? What would it be like to live inside a three-dimensional sphere? Can one continuously deform a trefoil knot to get its mirror image? Can the wind be blowing at every point on the earth at once? Can you tell if a graph is planar? Can you tell if a knot is trivial? Is there a list of all possible two-dimensional surfaces?

Contents

1. Relative homology
2. The exact homology sequences
3. Excion theorem & application to spheres
4. Mayer Victoris sequences
5. Jordan-Brouwer separation theorem
6. Spherical complexes
7. Betti number
8. Euler characteristic
9. Cell Complexes & adjunction spaces

Recommended Texts

1. Kosniowski, C. A. (1980). *First course in algebraict topology*. Cambridge: Cambridge University Press.
2. Greenberg, M. J., & Harper, J. R. (1967). *Algebraic topology, a first course.* Colorado: Westview Press.

Suggested Readings

1. Croom, F. H. (1978). *Basic concepts of algebraic theory*. Berlin: Springer.
2. Adhikari, M. R. (2016). *Basic algebraic topology & its applications* (1st ed.). Berlin: Springer.
3. Hatcher, A. (2001). *Algebraic topology*. Cambridge: Cambridge University Press.

MATH-6228 **Category Theory-I** **3(3+0)**

This is the first part of the two-course series Category Theory. Category Theory is a mathematical language & a toolbox that can be used for formalizing concepts that arise in our day-to-day activity. Category Theory focuses especially on the relations between the objects of interest & on different construction principles for objects. The aim of the course is to teach the language of category theory. This language is used throughout most of modern mathematics, & very convenient to express mathematical concept. The course will be filled with many examples from different branches of mathematics. If time will allow, we will also discuss category theory as a theory by itself & not only as a language and give some of it applications. Upon successful completion of this course students should be able to: Get familiar with the language of category theory & be able to use it in different branches of mathematics.

Contents

1. Definition & examples of category
2. Epimorphism
3. Monomorphism
4. Retractions
5. Initial, Terminal, & null objects
6. Category of graphs
7. Limits in categories
8. Equalizers
9. Pull backs
10. Inverse images
11. Intersections
12. Constructions with kernel pairs
13. Functions & adjoint Functions
14. Functions
15. Bifunctions
16. Natural transformations
17. Diagrams, Limits, Co limits
18. Universal problems & adjoint functions
19. Context-free Grammars & Context-free languages

Recommended Texts

1. Oosten, J. V. (2007). *Basic category theory*. Utrecht: University of Utrecht.
2. Rydeheard, D. E., & Burstall, R. M. (2001). *Computational category theory*. New Jersey: Prentice Hall.

Suggested Readings

1. Arbib, M. A., & Manes, E. G. (1977). *Arrows, structure & functions*. Massachusetts: Academic Press.
2. Herrlich, H., & Strecker, G. E. (1973). *Category theory*. Boston: Allyn & Bacon Inc.
3. Ehrig H., & Fender P. *Kategorien & automation.* Berlin: Walter de Gruyter.

MATH-6229 **Category Theory-II** **3(3+0)**

This course is a continuation of Category Theory-I. Category Theory is a mathematical language & a toolbox that can be used for formalizing concepts that arise in our day-to-day activity. The Category Theory focuses especially on the relations between the objects of interest & on different construction principles for objects. The aim of the course is to teach the language of the category theory. This language is used throughout the most of modern mathematics, & very convenient to express mathematical concept. The course will be filled with many examples from different branches of mathematics. If time will allow, we will also discuss category theory as a theory by itself & not only as a language & give some of it applications. Upon successful completion of this course students should be able to: Get familiar with the language of category theory & be able to use it in different branches of mathematics.

Contents

1. Subjects
2. Quotient objects & factorization
3. (E,M) Categories, (Epi external mono) & (external epi mono) Categories
4. (Generating external mono) factorization
5. Normal & exact categories
6. Additive categories
7. Abelian categories
8. Definition of automation & examples
9. Category of automata
10. Epimorphism
11. Monomorphism
12. Initial, terminal & null objects in Aut
13. Congruences
14. Factor automata
15. Automata with constant input & output

Recommended Texts

1. Oosten, J. V. (2007). *Basic category theory*. Utrecht: University of Utrecht.
2. Rydeheard, D. E., & Burstall, R. M. (2001). *Computational category theory*. New Jersey: Prentice Hall.

Suggested Readings

1. Arbib, M. A., & Manes, E. G. (1977). *Arrows, structure & functions*. Massachusetts: Academic Press.
2. Herrlich, H., & Strecker, G. E. (1973). *Category theory*. Boston: Allyn & Bacon Inc.
3. Ehrig, H., & Fender P. *Kategorien & automation.* Berlin: Walter de Gruyter.

MATH-6230 **Rings & Fields** **3(3+0)**

This course will cover basics of abstract rings and fields, which are an important part of any abstract algebra course sequence. We will begin with definitions and important examples. We will focus cover prime, maximal ideals and important classes of rings like integral domains, UFDs and PIDs. We will also prove the Hilbert basis theorem about noetherian rings. The last 3-4 weeks will be devoted to field theory. We will give definitions, basic examples. Then we discuss extension of fields, adjoining roots, and prove the primitive element theorem. Finally, we will classify finite fields. Rings are one of the fundamental languages of mathematics & they play a key role in many areas, including algebraic geometry, number theory, Galios theory & representation theory. The aim of this module is to give an introduction to rings. The imphasis will be on intresting examples of rings & their properties. This course introduces concepts of ring theory. The main objective of this course is to prepare students for courses which require a good background in Ring theory, Ring Homomorphism, basics Theorem etc. The focus of this course is the study of ideal theory & several domains in ring theory.

Contents

1. Definition of ring & basic concepts
2. Homomorphism theorems
3. Polynomial rings
4. Quotient rings
5. Unique factorization domain
6. Irreducibility of polynomials over UFD
7. Principal ideal domain
8. Factorization theory
9. Noetherian
10. Artinian rings
11. Euclidean domain
12. Arithmetic in Euclidean domain
13. Extension fields
14. Algebraic elements
15. transcendental elements
16. Simple extension

Recommended Texts

1. Cohn, P. M. (2006). *Free ideal rings & localization in general.* Cambridge: Cambridge University Press.
2. Lang, S. (2005). *Algebra*. Boston: Addison Wesley.

Suggested Readings

1. Herstein, I. N. (1975). *Topics in algebra*. New York: John Wiley & Sons Inc.
2. Hartley, B., & Hawkes, T. O. (1970). *Ring, modules & linear algebra*. Florida: Chapman & Hall
3. Fraleigh, J. A. (1982). *A first course in abstract algebra.* Boston: Addison Wesley.
4. Roman, S. (2005). *Field theory: Graduate texts in mathematics* (2nd ed). Berlin: Springer.

MATH-6231 **Theory of Modules** **3(3+0)**

This course is an introduction to module theory, who knows something about linear algebra and ring theory. Its main aim is the derivation of the structure theory of modules over Euclidean domains. This theory is applied to obtain the structure of abelian groups and the rational canonical and Jordan normal forms of matrices. The basic facts about rings and modules are given in full generality, so that some further topics can be discussed, including projective modules and the connection between modules and representations of groups. It aims to develop the general theory of rings & then study in some detail a new concept, that of a module over a ring. The theory of rings & module is key to many more advanced algebra courses. This subject presents the foundational material for the last of the basic algebraic structure pervading contemporary pure mathematics, namely fields & modules. The basic definitions & elementary results are given, followed by two important applications of the theory. This course introduces concepts of modules. The main objective of this course is to prepare students for courses which require a good background in Modules Theory, Primary component & Invariance Theorem etc.

Contents

1. Elementary notions & examples
2. Modules
3. Submodules
4. Quotient modules
5. Finitely generated
6. Cyclic modules
7. Exact sequences & elementary notions of homological algebra, Noetherian
8. Artinian rings & modules
9. Radicals
10. Semisimple rings & modules
11. Tensor product of modules
12. Bimodules
13. Algebra & coalgebra
14. Torsion module
15. Primary components
16. Invariance theorem

Recommended Texts

1. Adamson, J. (1976). *Rings & modules* (1st ed.). Chelsea. Prentice-Hall Inc
2. Blyth, T. S. (1977). *Module theory (*1st ed.). Oxford: Oxford University Press.

Suggested Readings

1. Hartley, B., & Hawkes, T.O. (1980). *Rings, modules & linear algebra (*1st ed.). Florida: Chapman & Hall.
2. Herstein, I. N. (1995). *Topics in algebra with application (*3rd ed.). New York: Books/Cole.
3. Jacobson, N. (1989). *Basic algebra, Vol. II (*2nd ed.). New York: Freeman.
4. Wang, F., & Kim, H. (2016). *Foundations of commutative rings & their modules* (1st ed.). Berlin: Springer.

MATH-6232 **Lie Algebra** **3(3+0)**

One important aspect of the study of lie algebra is the study of their representation. Although Ado's theorem is an important result, the primary goal of representation theory is not to find a faithful representation of a given lie algebra. In this course, Lie algebras are related to Lie groups, & both concepts have important applications to geometry & physics. Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, & describe parts of the classification mentioned above, especially the parts concerning root systems & Dynkin diagrams.The Lie algebras considered in this course will be finite dimensional vector spaces over endowed with a multiplication which is almost never associative are different in general.Students will learn how to utilize various techniques for working with Lie algebras, & they will gain an underst&ing of parts of a major classification result.

Contents

1. General Theory
2. Definitions & First Examples
3. Ideals & homomorphisms
4. Isomorphism Theorems
5. Lie algebra of derivations, Nilpotent Lie Algebras
6. Engel's Theorem, Solvable Lie Algebras
7. Lie's Theorem, Radical
8. Semi-simplicity, Killing form
9. Cartan's Criterion, Jordan
10. Chevalley Decomposition
11. Representations, Inner derivations
12. Course Units, Maximal ToralSubalgebras & Roots
13. Orthogonality Properties, Integrality Properties, Classification
14. Simple lie algebras & irreducible root systems
15. The Lie Algebra of Type G2 & Octonions
16. Representations Conjugacy theorems

Recommended Texts

1. Hall, B. C. (2015). *Lie groups, lie algebras, & representations* (2nd ed.). Berlin: Springer.
2. Erdmann, K., Wildon, M. J. (2006). *Introduction to lie algebras* (1st ed.). Berlin: Springer-Verlag.

Suggested Readings

1. Kirillov, J. (2008). *An introduction to lie groups & lie algebras* (1st ed.). Cambridge: Cambridge University Press.
2. Jacobson, N. (1979). *Lie algebra* (1st ed.). New York: Dover Publications.
3. Humphreys, J. E. (1994). *Introduction to lie algebra & representation theory* (1st ed.). Berlin: Springer-Verlag.

MATH-6233 **Advanced Functional Analysis** **3(3+0)**

This course is intended both for continuing mathematics students & for other students using mathematics at a high level in theoretical physics, engineering & information technology, & mathematical economics. This course introduces concepts of Fundamental Theorems & Spectral Theory. On satisfying the requirements of this course, students will have the knowledge & skills to explain the fundamental concepts of functional analysis & their role in modern mathematics & applied contexts. Moreover, it demonstrates accurate & efficient use of functional analysis techniques & the capacity for mathematical reasoning through analyzing, proving & explaining concepts from functional analysis. This course will mostly deal with the analysis of unbounded operators on a Hilbert or Banach space with a particular focus on Schrodinger operators arising in quantum mechanics. All the abstract notions presented in the course will be motivated & illustrated by concrete examples. In order to be able to present some of the more interesting material, emphasis will be put on the ideas of proofs & their conceptual underst&ing rather than the rigorous verification of every little detail.

Contents

1. Zorn’s lemma
2. Statement of Hahn-Banachtheorem for real vector spaces
3. Hahn-Banach theorem for complex vector spaces
4. Hahn-Banach theorem for normed spaces
5. Uniform boundedness theorem
6. Open mapping theorem
7. Closed graph theorem
8. Spectral properties of bounded linear operations on Normed Spaces
9. Further properties of Resolvent & spectrum
10. Use of complex Analysis in spectral theory
11. Compact linear operators on Normed Spaces

Recommended Texts

1. Kreyszig, E. (1989). *Introductory functional analysis with applications* (1st ed.). New York: John Wiley & Sons Inc.
2. Brown, A. L. (1970). *Elements of functional analysis* (1sted.). New York: Von Nostr& & Reinhold Company.

Suggested Readings

1. Oden, J. T. (1979). *Applied functional analysis* (1sted.). New Jersey: Prentice-Hall Inc.
2. Brown, A. L. (1970). *Elements of functional analysis*(1sted.). New York: Von Nostrand & Reinhold Company.

MATH-6234 **Galois Theory** **3(3+0)**

This module is designed for students in their third year & aims to introduce the basic concepts & techniques of Galois theory, building on earlier work at level 2. As such, it will provide an introduction to core concepts in rings, fields, polynomials & certain aspects of group theory. The word "algebra" means many things. The word dates back about 1200 years ago to part of the title of al-Khwarizmi's book on the subject, but the subject itself goes back 4000 years ago to ancient Babylonia & Egypt.Modern algebra is a cornerstone of modern mathematics. This course introduces concepts of ring & group theory. The main objective of this course is to prepare students for courses which require a good background in Group Theory, Rings, Galois Theory, Symmetric group & permutation group etc. It is assumed that the students possess some mathematical maturity & are comfortable with writing proofs.

Contents

1. Finite fields
2. Fields extension
3. Galois theory
4. Galois theory of equations
5. Construction with straight-edge & compass
6. Splitting field of polynomials
7. The galois groups
8. Some results on finite groups
9. Symmetric group as Galois group
10. Constructable regular n-gones
11. The Galois group as permutation group

Recommended Texts

1. Nicholson, W. K. (1993). *Introduction to abstract algebra* (1st ed.) PWS-Kent Publishing Co.
2. Ames, D. B. (1968). *Introduction to abstract algebra* (1st ed.). Pennsylvania: International textbook company.

Suggested Readings

1. Northcott, D. D. (1973). *A first course of homological algebra* (1st ed.). Cambridge: Cambridge University Press.
2. Jacobson, N. (1985). *Basic algebra I* (1sted.). New York: Freeman & Co.
3. Malik, D. S., Mordeson, J. N., & Sen, M. K. (1997). *Fundamentals of abstract Algebra*. New York: McGraw-Hill.

MATH-6235 **Measure Theory** **3(3+0)**

To introduce the concepts of measure & integral with respect to a measure, to show their basic properties, & to provide a basis for further studies in Analysis, Probability, & Dynamical Systems. To construct Lebesgue's measureon the real line & in n-dimensional Euclidean space. Measure theory provides a foundation for many branches of mathematics such as harmonic analysis, ergodic theory, theory of partial differential equations & probability theory. It is a central, extremely useful part of modern analysis, & many further interesting generalizations of measure theory have been developed. It is also subtle, with surprising, sometimes counter-intuitive, results. The aim of this course is to learn the basic elements of Measure Theory, with related discussions on applications in probability theory. Upon completion of this course student able to Transferable skills: Ability to use abstract methods to solve problems. The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs himself, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

1. Lebesgue measure
2. Introduction
3. Outer measure
4. Measurable sets
5. Lebesgue measure
6. A non-measurable set
7. Measurable functions
8. The Lebesgue integral of a non-negative function
9. The general Lebesgue integral
10. General measure & integration measure spaces
11. Measurable functions, integration
12. General convergence theorems

Recommended Texts

1. Roydon, H. L., & Fitzpatrick, P. M. (2017). *Real analysis* (4th ed.). New York: Collier Macmillan Co.
2. Cohn, D. L. (2013). *Measure theory* (2nd ed.). Bessel: Birkhauser.

Suggested Readings

1. Bartle, R. G. (1995). *The elements of integration & Lebasque measure*. New York: Wiley-Inter science.
2. Rudin, W. (1987). *Real & complex analysis* (3rd ed.). New York: McGraw Hill.
3. Cohn, D. L. (2013). *Measure theory* (2nd ed.). Bessel: Birkhauser.
4. Kubrusly, C. S. (2015). *Essentials of measure theory*. Berlin: Springer.

MATH- 6236 **Fluid Mechanics-I** **3(3+0)**

The course of fluid mechanics is introducing fundamental aspects of fluid flow behavior. Students will learn properties of Newtonian fluids; apply concepts of mass, momentum & energy conservation to flows. Fluid mechanics is the branch of physics concerned with the mechanics of fluids (liquids, gases, & plasmas) & the forces on them. It has applications in a wide range of disciplines,including mechanical, civil, chemical & biomedicalengineering, geophysics, oceanography meteorology, astrophysics, & biology. The course of fluid mechanics is introducing fundamental aspects of fluid flow behavior. Students will learn properties of Newtonian fluids; apply concepts of mass, momentum & energy conservation to flows. . The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs himself, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

1. Introduction: Definition of Fluid, basics equations
2. Methods of analysis, dimensions & units. Fundamental concepts
3. Fluid as a continuum, velocity field, stress field, viscosity, surface tension
4. Description & classification of fluid motions
5. Fluid Statics: The basic equation of fluid static
6. The st&ard atmosphere, pressure variation in a static fluid
7. Fluid in rigid body motion.
8. Basic equation in integral form for a control volume,
9. Basic laws for a system
10. Relation of derivatives to the control volume formulation
11. Conservation of mass, momentum equation for inertial control volume
12. Momentum equation for control volume with rectilinear acceleration
13. Momentum equation for control volume with arbitrary acceleration
14. The angular momentum principle
15. The first law of thermodynamics

Recommended Texts

1. Fox, R. W., & McDonald A. T. (2004). *Introduction to fluid mechanics (*6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: McGraw Hill.

Suggested Readings

1. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). New York: Dover Publications Inc.
2. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey.
3. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Cambridge: Cambridge University Press.

MATH-6237 **Fluid Mechanics-II** **3(3+0)**

This course is the seconed part of the core level course on fluid mechanics. This course covers properties of fluids, laws of fluid mechanics & energy relationships for incompressible fluids Studies flow in closed conduits, including pressure loss, flow measurement, pipe sizing & pump Selection, momentum equation for frictionless flow, Euler's equations, Bernoulli equation- Integration of Euler's equation, laminar flow & Boundary layers. . The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs himself, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

1. Incompressible inviscid flow
2. Momentum equation for frictionless flow
3. Euler's equations
4. Euler's equations in streamline coordinates
5. Bernoulli equation- Integration of Euler's equation along a streamline for steady flow
6. Relation between first law of thermodynamics & the Bernoulli equation
7. Unsteady Bernoulli equation-Integration of Euler's equation along a streamline
8. Irrotational flow. Internal incompressible viscous flow
9. Fully developed laminar flow
10. Fully developed laminar flow between infinite parallel plates
11. Fully developed laminar flow in a pipe
12. Part-B Flow in pipes & ducts
13. Shear stress distribution in fully developed pipe flow
14. Turbulent velocity profiles in fully developed pipe flow
15. Energy consideration in pipe flow
16. External incompressible viscous flow
17. Boundary layers, the boundary concept, boundary thickness, laminar flat plate
18. Boundary layer: exact solution, momentum, integral equation, use of momentum integral
19. equation for flow with zero pressure gradient
20. Pressure gradient in boundary-layer flow

Recommended Texts

1. Fox, R. W., & McDonald A. T. (2004). *Introduction to fluid mechanics (*6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: McGraw Hill.

Suggested Readings

1. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). New York: Dover Publications Inc.
2. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey.
3. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Cambridge: Cambridge University Press.

MATH-6238 **Quantum Mechanics–I** **3(3+0)**

This course is the first part of a two-course sequence., which covered most of the basic topics in quantum mechanics, including perturbation theory, operator techniques, and the addition of angular momentum. Quantum mechanics (QM; also known as quantum physics, quantum theory, the wave mechanical model & matrix mechanics), part of quantum field theory, is a fundamental theory in physics. It describes physical properties of nature on an atomic scale. Classical physics, the description of physics that existed before the theory of relativity & quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic & subatomic) scales, for which classical mechanics is insufficient. This course will introduce Dirac's braket formulation of quantum mechanics & make students familiar with various approximation methods applied to atomic, nuclear & solid-state physics, & to scattering.

Contents

1. Inadequacy of classical mechanics
2. Black body radiation, photoelectric effect
3. Compton effect
4. Bohr’s theory of atomic structure
5. Wave-particle duality
6. The de-Broglie postulate
7. The uncertainty principle
8. Uncertainty of position & momentum
9. Statement & proof of the uncertainty principle
10. Energy-time uncertainty
11. Eigenvalues & eigenfunctions
12. Operators & eigenfunctions
13. Linear operators, operator formalism in quantum mechanics
14. Orthonormal systems
15. Hermitian operators & their properties
16. Simultaneous eigenfunctions
17. Parity operators, postulates of quantum mechanics
18. The Schrödinger wave equation
19. Motion in one dimension
20. Step potential, potential barrier, potential well, & harmonic oscillator

Recommended Texts

1. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New York: George Allen & Unwin.
2. Powell, T. L., & Crasemann, B. (1961). *Quantum mechanics* (1st ed.). Boston: Addison Wesley.

Suggested Readings

1. Merzdacker, E. (1988). *Quantum mechanics* (1st ed.). New York: Wiley.

MATH-6239 **Quantum Mechanics–II** **3(3+0)**

This course is the second part of a two-course sequence. The primary goal of this course is to develop an understanding of some of the more advanced topics and techniques used in quantum mechanics. Most of this material will be essential for graduate research in many areas of physics, such as quantum optics, astrophysics, and atmospheric physics. This course will provide the necessary knowledge and skills to apply advanced techniques in quantum mechanics throughout the students' careers. Quantum mechanics (QM; also known as quantum physics, quantum theory, the wave mechanical model & matrix mechanics), part of quantum field theory, is a fundamental theory in physics. It describes physical properties of nature on an atomic scale. Classical physics, the description of physics that existed before the theory of relativity & quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic & subatomic) scales, for which classical mechanics is insufficient. This course is continuation of Quantum Mechanics-I & cover more advance topics.

Contents

1. Motion in three dimensions
2. Angular momentum
3. Commutation relations between components of angular momentum
4. Representation in spherical polar coordinates
5. Simultaneous Eigen functions of Lz & L2
6. Spherically symmetric potential & the hydrogen atom
7. Scattering theory
8. The scattering cross-section
9. Scattering amplitude
10. Scattering equation
11. Born approximation
12. Partial wave analysis
13. Perturbation Theory
14. Time independent perturbation of non-degenerate & degenerate cases
15. Time-dependent perturbations
16. Identical Particle: Symmetric & anti-symmetric Eigen function
17. The Pauli exclusion principle

Recommended Texts

1. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New York: George Allen & Unwin.
2. Powell, T. L., & Crasemann, B. (1961). *Quantum mechanics* (1st ed.). Boston: Addison Wesley.

Suggested Readings

1. Merzdacker, E. (1988). *Quantum mechanics* (1st ed.). New York: Wiley.
2. Taylor, G. (1970). *Quantum mechanics* (1st ed.). New York: George Allen & Unwin.

MATH-6240 **Electromagnetic Theory-I** **3(3+0)**

Electromagnetism is a branch of physics involving the study of the electromagnetic force, a type of physical interaction that occurs between electrically charged particles. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of coulomb's law, magnetic shells, conductivity & current density vector to flows. Students will learn properties of coulomb's law, magnetic shells, conductivity & current density vector to flows. It describes physical properties of nature on an atomic scale.

Contents

1. Electrostatics: Coulomb's law
2. Electric field & potential. lines of force & equipotential surfaces
3. Gauss's law & deduction
4. Conductor condensers
5. Dipoles, forces dipoles
6. Dielectrics, polarization & apparent charges
7. Electric displacement
8. Energy of the field, minimum energy
9. Magnetostatic field
10. The magnetostatic law of force, magnetic shells
11. Force on magnetic doublets
12. Magnetic induction, paradia & magnetism
13. Steady & slowly varying currents
14. Electric current
15. Linear conductors
16. Conductivity
17. Resistance
18. Kirchoff's laws
19. Heat production

Recommended Texts

1. Ferraro, V. C. A. (1956). *Electromagnetic theory* (Revised ed.). London: The Athlon Press
2. Reitz, J. R., Milford, F. J., & Christy, R. W. (1960). *Foundations of electromagnetic theory* (3rd ed.). Boston: Addison-Wesley.

Suggested Readings

1. Pugh, M. E. (196). *Principles of* electricity *& magnetism* (1st ed.). Boston: Addison-Wesley.
2. Powell, T. L., & Crasemann, B. (1961). *Quantum mechanics* (1st ed.). Boston: Addison Wesley.

MATH-6241 Electromagnetic Theory-II 3(3+0)

This course is the continuation of the course Electromagnetism-I. The classical (non-quantum) theory of electromagnetism was first published by James Clerk Maxwell in his 1873 textbook A Treatise on Electricity and Magnetism. A host of scientists during the nineteenth century carried out the work that ultimately led to Maxwell's electromagnetism equations, which is still considered one of the triumphs of classical physics. Maxwell's description of electromagnetism, which demonstrates that electricity and magnetism are different aspects of a unified electromagnetic field, holds true today. Electromagnetism is a branch of physics involving the study of the electromagnetic force, a type of physical interaction that occurs between electrically charged particles. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of simple introduction to Legendre polynomials, method of images, images in a plane, images with spheres & cylinders.

Contents

1. Vector potential
2. Forces on a circuit in magnetic field
3. Magnetic field energy
4. Law of electromagnetic induction
5. Co-efficient of self & mutual induction, Alternating current
6. Simple I.C.R circuits in series & parallel
7. Power factor, the equations of electromagnetism
8. Maxwell’s equations in free space & material media
9. Solution of Maxwell’s equations
10. Plane electromagnetic waves in homogeneous & isotropic media
11. Reflection & refraction of plane waves
12. Wave guides Laplace’ equation in plane
13. Polar & cylindrical coordinates

Recommended Texts

1. Ferraro, V. C. A. (1956). *Electromagnetic theory* (Revised ed.). London: The Athlon Press
2. Reitz, J. R., Milford, F. J., & Christy, R. W. (1960). *Foundations of electromagnetic theory* (3rd ed.). Boston: Addison-Wesley.

Suggested Readings

1. Pugh, M. E. (196). *Principles of electricity & magnetism* (1st ed.). Boston: Addison-Wesley.
2. Ferraro, V. C. A. (1956). *Electromagnetic theory* (Revised ed.). London: The Athlon Press

MATH-6242 Special Relativity 3(3+0)

Calculus Vector transformations Tensors for GTR to understand why we need these two theories. For that see the problems with Galilean transformation & equivalence of inertial & gravitational mass. The most important thing to study SR is to accept geometry as the concept behind it. The math is not difficult; it's the way of thinking you have to adopt. Draw space time diagrams, something to transform to another frame of reference (Lorentz transforms are available). Keep in mind that the view in the other reference frame is just a different view of the same situation that nothing really has changed, even if it looks different on euclidean paper. The electromagnetic force is carried by electromagnetic fields composed of electric fields & magnetic fields, & it is responsible for electromagnetic radiation such as light. It is one of the four fundamental interactions (commonly called forces) in nature, together with the strong interaction, the weak interaction, & gravitation. At high energy the weak force & electromagnetic force are unified as a single electroweak force. Students will learn properties of simple introduction to Legendre polynomials, method of images, images in a plane, images with spheres & cylinders.

Contents

1. Historical background & fundamental concepts of special theory of relativity
2. Michelson-Morley Experiment
3. Galilean transformations
4. Lorentz transformations (for motion along one axis)
5. Einstein Postulate of Special Theory of Relativity. Principle of Relativity & Constancy of velocity of light
6. Properties of Lorentz transformations
7. Length contraction
8. Time dilation, Relativity of simultaneity
9. Transformation law for velocity & acceleration
10. Vector form of General Lorentz transformations
11. Minkowski's Four-dimensional space-time world
12. Proper time
13. Introduction to 4-vector formalism. Lorentz transformations in 4-vector formalism
14. Lorentz transformation as rotation in Four-dimensional space

Recommended Texts

1. Puri, S. P. (2013). *Special theory of relativity*. New Delhi: Dorling Kindersley (India) Pvt. Ltd.
2. Qadir. (1989). *An introduction to the special relativity theory* (1st ed.). Singapore: World scientific Press.

Suggested Readings

1. D'Inverno, R. (1992). *Introducing Einstein's relativity* (1st ed.). Oxford: Oxford University Press.
2. Pugh, M. E. (196). *Principles of electricity & magnetism* (1st ed.). Boston: Addison-Wesley.

MATH-6243 **Elasticity Theory** **3(3+0)**

The course will provide a basic treatment of the formulation of the linear elasticity theory & its application to problems of stress & displacement analysis. Elasticity theory is the mathematical framework which describes such a deformation. By elastic, we mean that the material rebounds to its original shape after the forces on it are removed; a rubber eraser is a good example of an elastic material. The objectives of this course are to introduce to the students the analysis of linear elastic solids under mechanical & thermal loads, to introduce the theoretical fundamentals & to improve the ability to use the principles of theory of elasticity in engineering problems. The students who successfully complete the course should be expert in using indicial notion, Cartesian tensor analysis, analysis of stress & deformation, basic filed equations of the linear elastic solids & to formulate a solution strategies of various boundary value problems.

Contents

1. Homogeneous Isotropic Bodies
2. Elastic Moduli of Isotropic Bodies
3. Equilibrium equation for an isotropic elastic solid
4. Dynamical equation of an isotropic elastic solid
5. Strain- energy function
6. Strain- energy connection with Hook's law
7. Uniqueness of the solution of the boundary value problems of elasticity
8. Saint-Verant's principle extension
9. Torsion
10. Flexure of Homogeneous bears
11. Variational methods

Recommended Texts

1. Sokolinikoff. (1950). *Mathematical theory of elasticity* (2nd ed.). New York: McGraw Hill.
2. Funk, Y. C. (2007). *Foundations of solid mechanics*. New Jersey: Prentice Hall.

Suggested Readings

1. Love, A. E. H. (1944). *A treatise on the mathematical theory of elasticity*. Cambridge: Cambridge University Press.
2. Sadd, N. H. (2005). *Theory applications & numeric.* Amsterdam: Elsevier.
3. Boresi, A. P. (2000). *Elasticity in engineering mechanics.* New York: Wiley.

MATH-6244 **Analytic Dynamics** **3(3+0)**

Analytical dynamics develops Newtonian mechanics to the stage where powerful mathematical techniques can be used to determine the behavior of many physical systems. The mathematical framework also plays a role in the formulation of modern quantum & relativity theories. Topics studied are the kinematics of frames of reference (including rotating frames), dynamics of systems of particles, Lagrangian & Hamiltonian dynamics & rigid body dynamics. The emphasis is both on the formal development of the theory & also use of theory in solving actual physical problems. In classical mechanics, analytical dynamics, or more briefly dynamics, is concerned with the relationship between motion of bodies & its causes, namely the forces acting on the bodies & the properties of the bodies, particularly mass & moment of inertia.Analytical dynamics develops Newtonian mechanics to the stage where powerful mathematical techniques can be used to determine the behavior of many physical systems. The mathematical framework also plays a role in the formulation of modern quantum & relativity theories.

Contents

1. Generalized coordinates
2. Constraints
3. Degree of freedom
4. Classification of Dynamical system
5. Principle of virtual work. D' Alembert principle
6. Lagrange's equation of motion
7. Kinetic energy in generalized coordinates
8. Ignorable coordinates
9. Conservation theorems
10. Hamilton's principle, Derivation of Hamilton's principle
11. Hamilton's Equations of motion
12. Principle of Least Action
13. Applications to Harmonic oscillator
14. Canonical transformations
15. Generating functions
16. Poisson & Lagrange brackets with application
17. Bilinear co-variant
18. Hamilton Jacobi equation

Recommended Texts

1 Aruldhas, G. (2016). *Classical mechanics.* Delhi: PHI Private limited.
2 Chorlton, F. (1965). *Textbook of dynamics.* New York: Van Nostrand.

Suggested Readings

1. Greenwood, D. T. (1965). *Classical dynamics.* New Jersey: Prentice Hall.
2. Chester, W. (1979). *Mechanics.* London: Allen & Unwin Ltd.

MATH-6245 **Astronomy-I** **3(3+0)**

All Physics & Astronomy courses are expected to incorporate critical thinking abilities, quantitative skills & communication skills as core objectives in their course material & course work. They will also be required to demonstrate knowledge, underst&ing & use of the principles of physics &/or astronomy. In addition, there are objectives specific to Physics, Mathematics & Astronomy discipline courses. Both our overall & course-specific learning objectives are listed below. Students are required to demonstrate: (1) Knowledge, underst&ing & use of the principles of physics &/or astronomy. (2) Ability to use reasoning & logic to define a problem in terms of principles of physics. (3) Ability to use mathematics & computer applications to solve physics &/or astronomy problems. (4) Ability to design &/or conduct experiments &/or observations using principles of physics &/or astronomy & physics or astronomical instrumentation. (5) Ability to properly analyze & interpret data & experimental uncertainty in order to make meaningful comparisons between experimental measurements or observation & theory.

Contents

1. Introduction to Astronomy
2. The great & small circles
3. Spherical angle
4. Spherical triangle
5. Applications to the earth
6. Longitude
7. Latitude
8. Horizontal
9. Equatorial systems of coordinates
10. Observer’s meridian
11. Diurnal motion
12. Circumpolar stars
13. Right ascension
14. The equation of time
15. Basics of spherical trigonometry
16. The celestial sphere

Recommended Texts

1. Roy, A. E. (1982). *Astronomy: Principles & practice* (1st ed.). Bristol: Adam Hilger Ltd.
2. Wooland, E. W., & Clemence, G. M. (1966). *Spherical astronomy* (1st ed.). Boston: Academic Press.

Suggested Readings

1. Smart, W. M. (1977). *Textbook on spherical astronomy* (1st ed.). Cambridge: Cambridge University Press.
2. Chorlton, F. (1965). *Textbook of dynamics.* New York: Van Nostrand.

MATH-6246 **Astronomy-II** **3(3+0)**

This course is the continuation of the course Astronomy-I. An objective of the Course is to provide new research students in astronomy with an introduction & overview to research topics of major current & future interest. This will allow students to acquire some underst&ing & appreciation of research fields in addition to those they will be investigating as part of their graduate studies. Another objective is to provide new students with the opportunity to interact with each other, & also with lecturers who are leaders in their research field. As well as providing an introduction to astronomy research, the Course includes talks on career advice, public engagement activities in astronomy (of particular importance due to the popularity of astronomy with the general public, including children), unconscious bias in science & gender issues. All Physics & Astronomy courses are expected to incorporate critical thinking abilities, quantitative skills & communication skills as core objectives in their course material & course work.

Contents

1. Introduction to celestial navigation on earth
2. Celestial sphere
3. Time-keeping system
4. Refraction
5. Parallax & triangulation
6. Aberration
7. Precession
8. Nutation
9. Tropical measurements
10. Magnitude systems
11. Naked Eye Observations
12. Observational techniques
13. Optics & telescopes
14. Radio telescopes
15. Doppler imaging

Recommended Texts

1. Roy, A. E. (1982). *Astronomy: Principles & practice* (1^{st} ed.). Bristol: Adam Hilger Ltd.
2. Wooland, E. W., & Clemence, G. M. (1966). *Spherical astronomy* (1^{st} ed.). Boston: Academic Press.

Suggested Readings

1. Smart, W. M. (1977). *Textbook on spherical astronomy* (1^{st} ed.). Cambridge: Cambridge University Press.
2. Roy, A. E. (1982). *Astronomy: Principles & practice* (1^{st} ed.). Bristol: Adam Hilger Ltd.

MATH-6247 **Operations Research-I** **3(3+0)**

This course is the 1st part of the course series on operation research. Operations research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. Operations Research studies analysis and planning of complex systems. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real-life decision-making problems. The purpose of the course is to provide students with the concepts and tools to help them understand the operations research and mathematical modeling methods and to understand different application areas of operations research like transportation problem, assignment model, sequencing models, dynamic programming, game theory, replacement models & inventory models.

Contents

1. Linear Programming
2. Formulation & graphical solution
3. Simplex method
4. M-technique
5. Two-phase technique
6. Special cases
7. Sensitivity analysis
8. The dual problem
9. Primal dual relationship
10. The dual simplex method
11. Sensitivity
12. Post optimal analysis
13. Transportation model
14. Northwest corner
15. Least cost
16. Vogel's approximation methods
17. The method of multipliers
18. The assignment models

Recommended Texts

1. Hamdy, A. T. (2010). *Operations research an introduction* (9th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1sted.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research* (1st ed.). Amsterdam: North Holl&.
2. Hillier, F. S., & Liebraman, G. J. (2000). *Operations research* (8th ed.). New York: CBS.

MATH-6248 Operations Research-II 3(3+0)

Operations research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. Disciplines that are similar to, or overlap with, operations research include statistical analysis, management science, game theory, optimization theory, artificial intelligence & network analysis. All of these techniques have the goal of solving complex problems & improving quantitative decisions. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real-life decision-making problems. Students would be able to identify & develop complecated operational research modals from the verbal description of the real system. The underst&ing of the mathematical tools that are needed to solve optimization problems would be increased. They would be analyzing the results & propose the theoretical language underst&able to decision making processes in Management Engineering.

Contents

1. Algorithm for cyclic network
2. Maximal flow problems
3. Matrix definition of LP- problems
4. Revised simplex methods
5. Bounded variables decompositions algorithm
6. Parametric linear programming
7. Application of integer programming
8. Cutting plane algorithm
9. Mixed fractional cut algorithm
10. Branch methods
11. Bound methods
12. Zero-one implicit enumeration
13. Element of dynamics programming
14. Problems of dimensionality
15. Solutions of linear program by dynamics programming

Recommended Texts

1. Hamdy, A. T. (2010). *Operations research an introduction* (9th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1sted.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research* (1st ed.). Amsterdam: North Holl.
2. Hillier, F. S., & Liebraman, G. J. (2000). *Operations research* (8th ed.). New York: CBS.

MATH-6249 **Methods of Optimization-I** **3(3+0)**

Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. The objective of this course is to make students acquire a systematic understanding of optimization techniques. The course will start with linear optimization (being the simplest of all optimization techniques) and will discuss in detail the problem formulation and the solution approaches. Then we will cover a class of nonlinear optimization problems where the optimal solution is also globally optimal, i.e. convex nonlinear optimization and its variants. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

1. Introduction to optimization
2. Review of related mathematical concepts
3. Unconstrained optimization
4. Conditions for local minimizers
5. One dimensional search methods
6. Gradient methods
7. Newton's method (analysis & modifications)
8. Conjugate direction methods
9. Quasi Newton method
10. Application to neural network
11. Single Neuron Training
12. Linear integer programming
13. Genetic algorithms
14. Real number genetic algorithm

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory,* (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific

MATH-6250 **Methods of Optimization-II** **3(3+0)**

This is continuation of Methods of Optimization I. Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. Students will learn the foundations of linear programming, properties of optimal solutions and various solution methods for optimizing problems involving a linear objective function and linear constraints. Students will be exposed to geometric, algebraic and computational aspects of linear optimization and its extensions. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

1. Non-linear constrained optimization
2. Problems with equality constraints
3. Problem Formulation
4. Tangent & Normal spaces
5. Lagrange condition
6. Second-order conditions
7. Problems with inequality constraints
8. Karush-Kuhn-Tucker Condition
9. Second-order conditions
10. Convex optimization problems
11. Convex functions
12. Algorithms for constrained optimization
13. Lagrangian algorithms

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory,* (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsiitsiklis, J. (1997). *Introduction to lineae optimization* (2nd ed.). Belmont: Athena Scientific.

MATH-6251 **Theory of Splines-I** **3(3+0)**

This is the first part of the two-course series of Theory of Splines. This course is designed to teach students about basics of scientific computing for solving problems which are generated by data using interpolation & approximation techniques & learn how to match numerical method to mathematical properties. This course gives the students the knowledge of problem classes, basic mathematical & numerical concepts & software for solution of engineering & scientific problems formulated as using data sets. After successful completion, students should be able to design, implement & use interpolations for computer solution of scientific problems involving problems generated by set of data. The material covered provides the studnets with the necessary tools for understanding the many applications of splines in such diverse areas as approximation theory, computer-aided geometric design, curve and surface design and fitting, image processing, numerical solution of differential equations, and increasingly in business and the biosciences.

Contents

1. Basic concepts of Euclidean geometry
2. Scalar & vector functions
3. Barycentric coordinates, Convex hull
4. Matrices of affine maps
5. Translation, rotation, scaling
6. Reflection & shear,
7. Curve fitting, least squares line fitting, Least squares power fit
8. Data linearization method for exponential functions, nonlinear least-squares method for exponential functions
9. Transformations for data linearization
10. linear least squares, Polynomial fitting,
11. Basic concepts of interpolation, Lagrange’s method, error terms & error bounds of Lagrange’s method, Divided differences method,
12. Newton polynomials, error terms & error bounds of Newton polynomials
13. central difference interpolation formulae
14. Gauss’s forward interpolation formula, Gauss’s backward interpolation formula
15. Hermite’s methods

Recommended Texts

1. David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science + Business Media Inc.
2. John, H. M., & Kurtis, D. F. (1999). *Numerical methods using MATLAB*. New Jersey: Prentice Hall.

Suggested Readings

1. Rao, S. S. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.
2. Sudaran, R. K. (1996). *A first course in optimization theory* (3rd ed.). Cambridge: Cambridge University Press.
3. Chang, E. K. P. & Zak, S. I. I. (2004). *An introduction to optimization* (3rd ed.). New York: Wiley.

MATH-6252 **Theory of Splines-II** **3(3+0)**

The goal of the course is to provide the students with a strong background on numerical approximation strategies & a basic knowledge on the theory of splines that supports numerical algorithms. Interactive graphics techniques for defining & manipulating geometrical shapes used in computer animation, car body design, aircraft design, & architectural design. In this course follow a modular approach & contribute different components to the development of an interactive curve & surface modeling system. Curve Modeling Techniques: Students will implement various curve interpolation & approximation techniques that allow the interactive specification of three-dimensional curves (e.g. Bezier, B-spline, rational curves). Surface modeling techniques: Students will implement various surface interpolation & approximation techniques that allow the interactive specification of three-dimensional surfaces (e.g. Bezier, B-spline, rational surfaces). Simple, 3D Modeling System: Students will integrate the curve & surface modules into a system that allows the user to interactively design & store simple, 3D geometries.

Contents

1. Parametric curves (scalar & vector case), Algebraic form
2. Hermite form, control point form, Bernstein Bezier form
3. Matrix forms of parametric curves
4. Algorithms to compute B.B. form, Convex hull property
5. Affine invariance property, Variation diminishing property
6. Rational quadratic form, Rational cubic form
7. Tensor product surface, B.B. cubic patch
8. Quadratic by cubic B.B. patch, B.B. quartic patch
9. Splines, Cubic splines
10. End conditions of cubic splines, Clamped conditions
11. Natural conditions, second derivative conditions
12. Periodic conditions, Not a knot conditions
13. General splines, Natural splines, Periodic splines
14. Truncated power function, Representation of spline in terms of truncated power functions
15. Odd degree interpolating splines

Recommended Texts

1. Farin, G. (2002). C*urves & surfaces for computer aided geometric design, a practical guide* (5th ed.). New York: Academic Press.
2. Faux, I. D., & Pratt, M. J. (1979). *Computational geometry for design & manufacture* (1st ed.). New York: Halsted Press.

Suggested Readings

1. Bartle, H. R., & Beatly, C. J. (2006). *An Introduction to spline for use in computer graphics & geometric modeling* (4th ed.). Massachusetts: Morgan Kaufmann.
2. Boor, C. D. (2001). *A practical guide to splines* (Revised ed.). New York: Springer Verlag.

MATH-6253 **Graph Theory** **3(3+0)**

The objective of this course is to understand & apply the fundamental concepts in graph theory, apply graph theory-based tools in solving practical problems & to improve the proof writing skills. Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among others fields, to solve problems that can be modelled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including the group theory, the matrix theory, the numerical analysis, probability, topology, & the combinatorics. Even though some of the problems in graph theory can be described in an elementary way, many of these problems represent a challenge to many researchers in mathematics. The main focus of this course is to understand & apply the fundamental concepts in graph theory. To apply graph theory-based tools in solving practical problems. To improve the proof writing skills.

Contents

1. Graphs & digraphs
2. Degree sequences
3. Paths
4. Cycles, cut-vertices, & blocks
5. Eulerian graph
6. Digraphs
7. Trees
8. Incidence matrix
9. Cut-matrix
10. Circuit matrix & adjacency matrix
11. Orthogonality relation
12. Decomposition
13. Euler formula
14. Planer graphs
15. Non-planer graphs
16. Mengers theorem
17. Hamiltonian’s graphs

Recommended Texts

1. Chartrand. G., Lesniak, L., & Zhang, P. (2010). *Graphs & digraphs* (5^{th} ed.). Florida: Chapman & Hall.
2. Ruohonen, K. (2013). *Graph theory* (translation by Janne Tamminen, Kung-Chung Lee & Robert Piché).http://math.tut.fi/~ruohonen/GT_English.pdf

Suggested Readings

1. Robin, J. W. (1996). *Introduction to graph theory* (4^{th} ed.). Boston: Addison Wesley.
2. Bondy, J. A., & Murty, S. U. R. (1976). *Graph theory with applications*. United States: The Macmillian Press Ltd.

MATH-6254 **Theory of Automata** **3(3+0)**

Automata theory is the study of abstract machines & automata, as well as the computational problems that can be solved using them. It is a theory in theoretical computer science. The word automata (the plural of automaton) comes from the Greek word αὐτόματα, which means "self-making". The major objective of automata theory is to develop methods by which computer scientists can describe & analyze the dynamic behavior of discrete systems, in which signals are sampled periodically. ... Inputs: assumed to be sequences of symbols selected from a finite set I of input signals. The aim is to introduce to the students to the foundations of computability theory. Other objectives include the application of mathematical techniques & logical reasoning to important problems, & to develop a strong background in reasoning about finite automata & formal languages. At the end of the course the students should be able to: define the notion of countable & uncountable set, define the various categories of languages & grammars, define various categories of automata, define the notion of computability & decidability, & reduce a problem to another(when possible) to develop proofs of decidability/undecidibility. The course introduces some fundamental concepts in automata theory & formal languages including grammar, finite automaton, regular expression, formal language, pushdown automaton, & Turing machine. Not only do they form basic models of computation, they are also the foundation of many branches of computer science, e.g. compilers, software engineering, concurrent systems, etc.

Contents

1. Regular expressions
2. Regular Languages
3. Finite Automata
4. Context-free Grammars
5. Context-free languages
6. Push down automata
7. Decision Problems
8. Parsing
9. Turing Machines

Recommended Texts

1. Martin, J. C. (2010). *Introduction to languages & theory of computation* (4th ed.). New York: Mc Graw Hill.
2. Michael S. (2013). *Introduction to the Theory of Computation* (3rd ed.). New York: Cengage Learning.

Suggested Readings

1. Cohen, D. I. A. (1996). *Introduction to computer theory* (2nd ed.). New York: Wiley.
2. Linz, P. (2017). *Introduction to Formal Languages & Automata* (6th ed.). New York: Jones & Barlett.

MATH-6255 **Control Theory** **3(3+0)**

This course is an introduction to analysis and design of feedback control systems, including classical control theory in the time and frequency domain. Modeling of physical, biological and information systems using linear and nonlinear differential equations. In control system engineering is a subfield of mathematics that deals with the control of continuously operating dynamical system in engineered processes & machines. The objective is to develop a control model for controlling such systems using a control action in an optimum manner without delay or overshoot & ensuring control stability. The course prepares students to do independent work at the frontiers of systems theory and control engineering. The course builds further on standard linear systems theory to explore issues related to optimization, estimation and adaptation. Students will learn to formulate and appreciate fundamental limitations in control, filtering and estimation.

Contents

1. System dynamics & differential equations, some system equations, system control
2. Mathematical methods & differential equations, The classical & modern control theory
3. Transfer functions & block diagram, Review of Laplace Transforms
4. Applications to differential equations, Transfer functions & Block diagrams
5. State space formations, State space forms, using transfer functions to define state variables
6. Direct solution of the state equation
7. Solutions of the state equation by Laplace transforms,
8. The transformation from companion to the diagonal state form
9. The transform function from the state equation, Transient & steady state response analysis
10. Response of first order system, Response of second order system, Response of higher order systems
11. Steady state error, Feedback control, The concept of stability
12. Routh stability criterion, Introduction to Liapunor’s method, Quadratic form
13. Determination of liapunov’s function,
14. The Nyquist stability criterion, The frequency response, an introduction to conformal mapping

Recommended Texts

1. Burghes, D., & Graham, A. (1980). *Introduction to control theory including optimal control*. New York: Ellis Horwood Ltd.
2. Cohen, D. I. A. (1996). *Introduction to computer theory* (2nd ed.). New York: Wiley.

Suggested Readings

1. Barnett, S., & Camron, R. G. (1985). *Introduction to mathematical control theory* (2nd ed.). Oxford: Oxford V. P.
2. Linz, P. (2017). *Introduction to Formal Languages & Automata* (6th ed.). New York: Jones & Barlett.

MATH-6256 **Applied Matrix Theory** **3(3+0)**

Matrix theory is a branch of mathematics which is focused on study of matrices. Initially, it was a sub-branch of linear algebra, but soon it grew to cover subjects related to graph theory, algebra, combinatorics & statistics as well. The aim of this course is to introduce the key mathematical ideas in matrix theory, which are used in modern methods of data analysis, scientific computing, optimization, & merely all quantitative fields of science & engineering. While the choice of topics is motivated by their use in various disciplines, the course will emphasize the theoretical & conceptual underpinnings of this subject, just as in other (applied) mathematics course. The main focus of this course is to study the basics of matrices & their applications. Moreover, it concerns with the variational principles, Weyl's inequalities, Gershgorin's theorem & perturbations of the spectrum. The aim of this course is to introduce the key mathematical ideas in matrix theory, which are used in modern methods of data analysis, scientific computing, optimization, & merely all quantitative fields of science & engineering. While the choice of topics is motivated by their use in various disciplines, the course will emphasize the theoretical & conceptual underpinnings of this subject, just as in other (applied) mathematics course.

Contents

1. Eigen values
2. Eigen vectors
3. The Jordan canonical forms
4. Bilinear & quadratic forms
5. Matrix analysis of differential equations
6. Variational principles
7. Perturbation theory
8. The Courant minimax theorem
9. Weyl's inequalities
10. Gershgorin's theorem
11. Perturbations of the spectrum
12. Vector norms & related matrix norms
13. The condition number of a matrix

Recommended Texts

1. Strang, G. (2005). *Linear algebra & its applications*. Cambridge: Academic Press.
2. William, G. (2009). *Linear algebra with applications* (7th ed.). Boston: Allyn & Bacon, Inc.

Suggested Readings

1. Stewart, G. W. (1973). *Introduction to matrix computations*. New York: Academic Press.
2. Franklin, J. N. (2000). *Matrix theory* (1st ed.). New York: Dover Publications.
3. Laub, A. J. (2005). *Matrix analysis for scientsts and engineers*. United States: SIAM.

MATH-6257 **Finite Element Analysis** **3(3+0)**

The objective of finite element method is to discretize the domain into finite element for which the governing equations are algebraic equations. Solution of these algebraic equations gives the approximate solution of the nonlinear differential equations. The convergence is judged by the refinement of mesh. To provide the fundamental concepts of the theory of the finite element method, to obtain an underst&ing of the fundamental theory of the FEA method, to develop the ability to generate the governing FE equations for systems governed by partial differential equations, to underst& the use of the basic finite elements for structural applications using truss, beam, frame, & plane elements; & to underst& the application & use of the FE method for heat transfer problems. This course also to develop proficiency in the application of the finite element method (modeling, analysis, & interpretation of results) to realistic engineering problems through the use of a major commercial general-purpose finite element code.

Contents

1. Rational Bezier curves
2. Properties of rational Bezier curves
3. Marsden identity
4. Construction of FEM basis function
5. The de Boor algorithm
6. Dual functional
7. Error approximation by orthogonal functional
8. Cubic Hermite interpolation
9. Natural spline interpolation
10. Quasi interpolant
11. Schoenberg scheme, error of quasi interpolation
12. Lagrangian function for interpolation, interpolation error
13. Curves on uniform grid & their properties, interpolation with curves on uniform grid
14. Geometric Hermite interpolation
15. Non-uniform rational B-splines
16. Construction of finite element basis on multidimensional space
17. Box splines
18. Recursion for Box splines
19. Approximation on multidimensional space
20. Ellipticity of approximation
21. Cea’s lemma
22. Approximation theorems for FEM

Recommended Texts

1. Andersson, L. E. (2010). *Introduction to the mathematics of subdivision surfaces.* Philadelphia: SIAM.
2. Quarteroni, A. (2009). *Numerical models for differential problems.* Berlin: Springer.

Suggested Readings

1. Bathe, K. J. (2007). *Finite element method.* New York: John Wiley & Sons Inc.

MPhil
MATHEMATICS

MATH-7101 **Representation Theory-I** **3(3+0)**

The aim of the course is to give an introduction to the theory of representation. The chief emphasis will be on the three areas: finite groups, compact Lie groups & complex Lie algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation. Representation theory is an area of mathematics which, roughly speaking, studies symmetry in linear spaces. It is a beautiful mathematical subject which has many applications, ranging from number theory & combinatorics to geometry, probability theory, quantum mechanics & quantum field theory. Representation theory was born in 1896 in the work of the German mathematician F. G. Frobenius. This work was triggered by a letter to Frobenius by R. Dedekind. In essence, a representation makes an abstract algebraic object more concrete by describing its elements by matrices & the algebraic operations in terms of matrix addition & matrix multiplication.

Contents

1. Preliminaries from group theory
2. Group representations
3. FG-modules
4. FG-sub-modules
5. Reducibility
6. Group algebras
7. FG-homeomorphisms
8. Maschke's Theorem
9. Schur's lemma
10. Irreducible modules & the group algebra
11. Conjugacy classes
12. Characters
13. Inner products of characters
14. The number of irreducible characters.

Recommended Texts

1. James, G., & Liebech, M. (2001). *Representations & characters of groups* (2nd ed.). Cambridge: Cambridge University Press.
2. Tullio, C. S., Fabio, S., & Filipoo, T. (2008). *Harmonic analysis on finite groups: representation theory, gelf & pairs & Ma rkov chains* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Charles, W. C., & Irving, R. (2006). *Representation theory of finite groups & associative algebras* (1st ed.). Michigan: American Mathematical Society.
2. William, F., & Young, T. (2012). *Young tableaux: with applications to representation theory & geometry* (3rd ed.). Cambridge: Cambridge University Press.
3. Recent research articles.

MATH-7102 **Semigroup Theory** **3(3+0)**

In some respects we can think of a semigroup as an abstraction of a group but on the other hand it is sometimes useful to compare the theory of semigroups with that of rings (the 'multiplicative part' of a ring is just a semigroup) & many of the historical developments in the theory of semigroups owe much to these two theories. However recent work has highlighted strong connections with, for example, many aspects of theoretical computer science (automata theory, theory of codes & formal language theory) as well as with other areas of mathematics such as the theory of ordered structures & (partial) symmetries. At the end of the course, students should be familiar with the basic ideas of the subject, including Green's relations, and be able to handle the algebra of semigroups in a comfortable way, the role of structure theorems, and be able to use Rees' theorem for completely 0-simple semigroups and have an appreciation of the place of semigroup theory in mathematics.

Contents

1. Semigroups: Introductory ideas & basic definitions
2. Cyclic semigroups, Ordered sets
3. Semi lattices
4. Binary relations
5. Equivalences congruences
6. Free semi-groups, Green's Equivalences
7. L,R,H,J & D Regular semi groups
8. O-Simple semigroups
9. Simple & O-Simple semi groups
10. Rees's theorem
11. Primitive idempotent
12. O-Simple semi groups
13. Finite congruence free semigroups
14. Union of groups, Bands
15. Free b&s varieties of b&s Inverse semigroups
16. Congruences on Inverse semigroups
17. Fundamental inverse semigroups
18. Bisimple & simple inverse semigroups
19. Orthodox semigroups, Basic properties
20. The structure of orthodox semigroups

Recommended Texts

1. Sinha, K., & Srivastava, S. (2017). *Theory of semigroups and applications* (1st ed.). Singapore: Springer.
2. Clifford, A. H., & Preston. G. B. (1967). *The algebraic theory of semigroups*. Vol. I & II. Michigan: AMS Math. Surveys.

Suggested Readings

1. Howie J. M. (1976). *An introduction to semigroup theory.* Cambridge: Academic Press.
2. Related Research Papers.

MATH-7103 Advanced Ring Theory-I 3(3+0)

The philosophy of this subject is that we focus on similarities in arithmetic structure between sets (of numbers, matrices, functions or polynomials for example) which might look initially quite different but are connected by the property of being equipped with operations of addition & multiplication. The set of integers & the set of 2 by 2 matrices with real numbers as entries are examples of rings. These sets are obviously not the same, but they have some similarities & some differences - in terms of their algebraic structure. Although people have been studying specific examples of rings for thous&s of years, the emergence of ring theory as a branch of mathematics in its own right is a very recent development. Much of the activity that led to the modern formulation of ring theory took place in the first half of the 20th century. Ring theory is powerful in terms of its scope & generality, but it can be simply described as the study of systems in which addition & multiplication are possible. A ring is an important fundamental concept in algebra & includes integers, polynomials & matrices. Ring theory has applications in number theory & geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. In this module we shall develop ring & module theory leading to the fundamental theorems of Wedderburn & some of its applications.

Contents

1. Radical classes
2. Semi-simple classes
3. The upper radical
4. Semi-simple images
5. The lower radical hereditariness of the lower radical class
6. The upper radical class
7. Partitions of simple rings

Recommended Texts

1. Huynh, D. V. (2010). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser.
2. López, P., Sergio, R., & Van, H., Dinh. (2010). *Advances in ring theory*. Switzerland: Birkhäuser.

Suggested Readings

1. Wiegandt, R. (1974). *Radical & semisimple classes of rings* (1st ed.). Ontario: Queen's University.
2. Jain, S. K., & Tariq, R. S. (1997). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser Basel.
3. Golan, J. S. (1999). *Semirings & their applications*. Netherlands: Springer.

MATH -7104 **Theory of Group Actions** **3(3+0)**

The purpose of this course is to give the introduction to theory of group actions. A group action is a representation of the elements of a group as symmetries of a set. the first group action studied was the action of the Galois group on the roots of a polynomial. However, there are numerous examples & applications of group actions in many branches of mathematics, including algebra, topology, geometry, number theory, & analysis, as well as the sciences, including chemistry & physics. Group theory is one of the great simplifying & unifying ideas in modern mathematics. It plays a role in our underst&ing of fundamental particles, the structure of crystal lattices & the geometry of molecules. In this course, we will begin by revising the simple axioms satisfied by groups & begin to develop basic group theory by reference to some elementary examples. We will also examine how the notion of a permutation group can be generalized to that of a group action on a set & will show how to use this in certain counting problems arising in combinatorics.

Contents

1. Preliminaries
2. The theory of group actions
3. Coset space
4. Multiplicative group of a finite fields
5. Extensions of finite fields
6. Projective line over finite fields
7. Projective
8. Linear groups through actions

Recommended Texts

1. Magnus, W., Karrass, A., & Solitar, D. (2004). *Combinatorial group theory* (Revised ed.). New York: Dover Publications.
2. Rose, J. S. (1994). *A Course on group theory* (1st ed.). New York: Dover Publications.

Suggested Readings

1. Coxeter, H. S. M., & Moser, W. O. J. (1980). *Generators & relations for discrete groups* (4th ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Joseph, G. (2012). *Contemporary abstract algebra* (8th ed.). New York: Brooks/Cole.
3. Allenby, R. B. J. T. (1991). *Rings, fields & groups* (2nd ed.). New York: Elsevier.
4. Recent research articles.

MATH -7105 **Graph Theory** **3 (3+0)**

Graph theory is one branch of the wide-ranging field known nowadays as combinatorics. It has applications in many different areas, including parts of computer science, operations research including scheduling, network flows & circuit design. Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among others fields, to solve problems that can be modeled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology, & combinatorics. Even though some of the problems in graph theory can be described in an elementary way, many of these problems represent a challenge to many researchers in mathematics. Graph theory plays an important role in operating system in solving job scheduling & resource allocation problems. The concept of graph coloring is applied in job scheduling problems of CPU. A graph is symbolic representation of a network & connectivity. It is concerned that how networks can be encoded.

Contents

1. Fundamental & basic definitions
2. Paths cycles & trees
3. Hamilton cycles & Euler circuits
4. Planer graphs. Flows
5. Connectivity & Matching Network flows
6. Connectivity & Menger’s theorem
7. External problems paths & Complete Subgraphs
8. Hamilton path & cycles
9. Colouring, Vertex colouring
10. Edge coloring
11. Graph on surfaces

Recommended Texts

1. Bondy, J. A., & Murty, U. S. R. (2010). *Graph theory with applications*. London: Macmillan.
2. Gross, J. L., & Yellen, J. (2005). *Graph theory & its applications*. Cambridge: CRC press.

Suggested Readings

1. Bollobás, B. (2013). *Modern graph theory* (Vol. 184). Berlin: Springer Science & Business Media.
2. Trudeau, R. J. (2013). *Introduction to graph theory*. Massachusetts: Courier Corporation.
3. Chartrand, G. (2006). *Introduction to graph theory*. New York: McGraw-Hill Education.
4. Golumbic, M. C. (2004). *Algorithmic graph theory & perfect graphs*. Netherlands: Elsevier.

MATH -7106 **Topological Vector Spaces** **3(3+0)**

The aim of this course is to provide an introduction to the theory of topological vector spaces, with a focus on locally convex spaces. Dual pairs of vector spaces & the weak topology are also introduced. The infinite-dimensional spaces are ubiquitous in many branches of mathematics, & their topologies are almost always interesting. Samples of spaces that one is sure to encounter are Cc(R), C∞(R), L p (R), & their continuous linear duals. We will start by briefly recalling the concepts of topology & continuity. This course contains the properties of the weak topology on Banach spaces, Krein's theorem on the closed convex hull of weakly compact sets in a the Banach space, the quotient spaces, Closed hyperplanes, completeness of topological vector spaces. Lastly, it addresses topics such as the locally convex final topology, Mertizable topological vector spaces, Normable topological vector spaces, Convex & compact sets in locally convex spaces.

Contents

1. Sub-linear functionals & extension of linear functional
2. Topological vector spaces: definitions & general properties
3. Quotient spaces, totally bounded sets
4. Convex sets & compact sets in topological vector spaces
5. Closed hyperplanes & separation of convex sets
6. Complete topological vector spaces
7. Mertizable topological vector spaces
8. Normed vector spaces
9. Normable topological vector spaces & finite dimensional spaces
10. Locally convex spaces
11. General properties
12. Subspaces, product spaces in locally convex spaces
13. Quotient spaces in locally convex spaces
14. Convex & compact sets in locally convex spaces

Recommended Texts

1. Voigt, J. (2020). *A course on topological vector spaces* (1st ed.). Switzerland: Birkhäuser Basel.
2. Narici, L., & Beckenstein, E. (2010). *Topological vector spaces* (2nd ed.). Florida: Chapman & Hall/CRC.

Suggested Readings

1. Cristescu, R. (1977). *Topological vector spaces* (1st ed.). Netherlands: Noordh off International Publishing.
2. Treves, F. (1967). *Topological vector spaces, distributions & kernels* (1st ed.). New York: Academic Press.

MATH -7107 Advanced Complex Analysis-I 3(3+0)

This course is intended both for mathematics students continuing to honours work & for other students using mathematics at a high level in theoretical physics, engineering & information technology, & mathematical economics. This course introduces the calculus of complex functions of a complex variable. It turns out that complex differentiability is a very strong condition & differentiable functions behave very well. Integration is along paths in the complex plane. The central result of this spectacularly beautiful part of mathematics is Cauchy's Theorem guaranteeing that certain integrals along closed paths are zero. This striking result leads to useful techniques for evaluating real integrals based on the 'calculus of residues. The course presents an introduction to some topics of contemporary complex analysis, in particular spaces of analytic functions, quasiconformal mappings, univalent functions. The purpose is to prepare the student to independent work in these topics & especially to use the methods of complex analysis in other areas of mathematics, (for example harmonic analysis & differential equations) as well as in applied areas (fluid dynamic, signal analysis, statistics). The content may vary, dependent on the needs & interests of the students.

Contents

1. Analytic continuation
2. Equation of continuity
3. Uniform boundedness
4. Normal & compact families of analytic functions
5. External problems
6. Harmonic functions & their properties
7. Green's & von Neumann functions & their applications
8. Harmonic measure conformal mapping
9. The Riemann mapping theorem
10. The Kernel function
11. Functions of several complex variables

Recommended Texts

1. Hille, E. (2006). *Analytic function theory*. New York: American Mathematical Society.
2. Schinznger. R., & Laura, P. A. A. (2003). *Conformal mapping: methods & applications*. New York: Dover Publications.

Suggested Readings

1. Nehari, Z. (1982). *Conformal mappings*. New York: Dover Publications.
2. Sansone, G., & Gerretsen, J. (1969). *Lectures on the theory of functions of a complex variable* (Vol. 2). Netherlands: Wolters-Noordhoff Publishing.
3. Recent research articles.

MATH-7108 **Fixed Point Theory** **3(3+0)**

One of the most dynamic area of research of the last 50 years, fixed point theory plays a fundamental role in several theoretical and applied areas, such as nonlinear analysis, integral and differential equations and inclusions, dynamic systems theory, mathematics of fractals, mathematical economics (game theory, equilibrium problems, optimization problems) and mathematical modeling. The theory of fixed points has been revealed as a major, powerful & important tool in the study of nonlinear phenomena. The aim of this course is to understand the theory of fixed point which is an important branch of nonlinear functional analysis. This subject gives sufficient directions to Acquiring general knowledge in & concepts of fixed point theory as well as enabling students to successfully apply it when needed in other courses. This course is intended as a brief introduction to the subject with a focus on Banach Fixed Point theorems fixed point theorem & its application to nonlinear differential equations, nonlinear integral equations, real & complex implicit functions theorems & system of nonlinear equations. Some generalizations & similar results e. g. Kannan Fixed Point theorems, Banach Fixed Point theorem for multi-valued mappings are also educated.

Contents

1. Fixed point: Definitions & examples
2. Fixed point iteration procedure
3. Picard iteration
4. Banach's contraction principle
5. Contractive mappings on metric spaces & related fixed point theorems
6. Non-expansive mappings
7. Sequential approximation techniques for non-expansive mappings
8. Properties of fixed point sets & minimal set
9. Multivalued mappings & related fixed point theorems
10. The Topological fixed point property & Brouwer's fixed point theorem
11. Applications of fixed point theorems

Recommended Texts

1. Granas, A., & Dugundji, J. (2010). *Fixed point theory* (1st ed.). New York: Springer.
2. Agarwal, R. P., Meehan, M., & Regan, D. O. (2009). *Fixed point theory & applications* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Almezel, S., Ansari, Q. H., & Khamsi, M. A. (2014). *Topics in fixed point theory* (1st ed.). New York: Springer.
2. Berinde, V. (2007). *Iterative approximation of fixed points* (2nd ed.). New York: Springer-Verlag Berlin.
3. Goebel, K., & Kirk, W. A. (1990). *Topics in metric fixed point theory* (1st ed.). New York: Cambridge University Press.

MATH -7109 Approximation Theory 3(3+0)

Approximation theory is the branch of mathematics which studies the process of approximating general functions by simple functions such as polynomials, finite elements or Fourier series. It therefore plays a central role in the analysis of numerical methods, in particular approximation of PDE’s. The chapter introduces various function spaces, in particular Hölder, Sobolev & Besov smoothness classes. Two important types of inequalities that are essential to build a more general theory are discussed: direct (or Jackson type) inequalities, & inverse (or Bernstein type) inequalities that take into account the smoothness properties of the V_j spaces. In the setting of finite element spaces, such inequalities are classically derived by using the properties of the affine mapping between each element & a reference domain & basic results of polynomial approximation in this reference domain. The primary objective of this course is to lay the theoretical foundation for the wider field of numerical analysis by looking at some of the classical topics in approximation theory. At the end of the course, the students are expected to underst& & master theoretical as well as practical issues that arise in approximation of functions by polynomials, trigonometric polynomials, splines & by rational functions

Contents

1. Best approximation in metric spaces
2. Best approximation in normed spaces
3. Least square approximation
4. Rational approximation
5. Haar condition in function spaces
6. Best approximation in function spaces
7. Interpolation stone
8. Weierstrass theorem for scalar valued functions
9. Weierstrass theorem for vector valued functions
10. Spline approximation

Recommended Texts

1. Achieser, N. I. (2004). *Theory of approximation*, New York: Dover Publications.
2. Cheney, W. E. (2000). *Introduction to approximation theory* (2^{nd} ed.). Providence: Amer Mathematical Society.

Suggested Readings

1. Powell, M. D. (1981). *Approximation theory & methods*, Cambridge: Cambridge University Press.
2. Holmes, R. B. (1971). *A course on optimization & best approximation, lecture notes in mathematics no. 257*, New York: Springer-Verlag.
3. Recent research articles.

MATH -7110 **Topological Algebras** **3(3+0)**

The aim of this course is to study the topological spaces strongly related to groups: either the spaces themselves are groups in a nice way (so that all the maps coming from group theory are continuous), or groups act on topological spaces & can be thought of as consisting of homeomorphisms. Groups are attributed to Algebra. In the mathematics built on sets, main objects are sets with additional structure such as topology, metric, partial order. Topology & metric evolved from geometric considerations. Algebra studied algebraic operations with numbers & similar objects & introduced into the set-theoretic Mathematics various structures based on operations. One of the simplest (& most versatile) of these structures is the structure of a group. It emerges in an overwhelming majority of mathematical environments. It often appears together with topology & in a nice interaction with it. This interaction is a subject of Topological Algebra. In this course, we will study some important topological algebras including Q- algebras, Semi-simple algebra & Involutive algebra.

Contents

1. Definition of Topological algebra & its examples
2. Adjumetion of unity
3. Locally convex algebras. Idempotent & m-convex sets
4. Locally multicatively convex (l.m.c) algebras
5. Q-algebras
6. Frechet algebras, Spectrum of an element, Spectral radius
7. Basic theorems on Spectrum, Gelf&-Mazur theorem
8. Maximal ideals, quotient algebras
9. Multiplicative linear functional & their continuity
10. Gelf& transformations
11. Radical of an algebra
12. Semi-simple algebras
13. Involutive algebra
14. Gelf&-Naimark theorem l.m.c algebras

Recommended Texts

1. Arhangel'skii, A., & Tkachenko, M. (2008). *Topological groups & related structures, an introduction to topological algebra* (1st ed.). Paris: Atlantis Press.
2. Mallios, A. (1986). *Topological algebras, selected topics* (1st ed.). Hoofddrop: North-Holland Company.

Suggested Readings

1. Hussain, T. (1983). *Multiplicative functions on topological algebras, research notes in mathematics.* Boston: Pitman Advanced Publishing Program.
2. Beckenstein, E., Narici, L., & uffel, C. (1977). *Topological algebras* (1st ed.). Hoofddrop: North-Holland Company.
3. Recent research articles.

MATH -7111 **Commutative Algebra-I** **3(3+0)**

This course will be an introduction to commutative algebra, a subject that has interactions with algebraic geometry, number theory, combinatorics, & several complex variables. In early classes, you learnt to solve systems of linear equations in many variables. But what about equations of higher degree? You probably have encountered a few methods so far to find the zeroes of univariate polynomials. But what about solutions to sets of polynomial equations in several variables? Such sets equations come up naturally in kinematics, robotics, physics, statistics, biology, optimization, etc. Commutative Algebra is the study of commutative rings, polynomial rings & their modules & ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry & algebraic number theory.

Contents

1. Commutative Rings: definition & examples
2. Integral domains, unit, irreducible & prime elements in ring
3. Types of ideals, Quotient rings, Rings of fractions, Ring homomorphism
4. Euclidean domains, principal ideal domains & unique factorization domains
5. Polynomial & Formal Power series Rings
6. Construction of formal power series ring R[[X]] & polynomial ring R[X] in one indeterminate,
7. Formal power series & polynomial rings in n indeterminate
8. Factorization in polynomial rings, irreducibility criteria,
9. Noetherian Rings: Definition & examples. Polynomial extension of Noetherian domains
10. Quotient ring of Noetherian rings, Ring of fractions of Noetherian rings,
11. Dimension of Rings: Chain of prime ideals in a domain, length of chain of prime ideals
12. Dimension of polynomial rings, Integral Dependence: Ring extension,
13. Integral element, almost integral element, integral closure of a domain
14. Complete integral closure of a domain, integrally closed & completely integrally closed domain,
15. Valuation Rings: definition & examples, Valuation map & value group, Rank of valuation
16. valuation map & valuation ring, valuation ring is integrally closed,
17. Discrete Valuation Rings & Dedekind domains: Fractional ideals,
18. Finitely generated fractional ideals, invertible fractional ideals, discrete valuation rings
19. Definitions & examples of Dedekind domains

Recommended Texts

1. Kemper, G. (2011). *A course in commutative algebra* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Atiyah, M. F., & Macdonald, I. G. (1969). *Introduction to commutative algebra* (1st ed.). Boston: Addison- Melbourne: Wesley Publishing Company.

Suggested Readings

1. Gilmer, R. (1972). *Multiplicative ideal theory* (1st ed.). New York: Marcell Dekker.
2. Matsumura, H. (1986). *Commutative ring theory* (1st ed.). Cambridge: Cambridge University Press.

MATH -7112 **Theory of Semirings** **3(3+0)**

This course will be an introduction to theory of semirings. Semiring theory stands with a foot in each of two mathematical domains. On one hand, semirings are the abstract mathematical structures & their study is part of abstract algebra arising ab initio from the work of Dedekind, Macaulay, krull, & others on the theory of ideals of a commutative ring & then through the more general work of V&iver & the tools used to study them are primarily the tools of abstract algebra. On the other, the modern interest in semirings arises primarily from fields of applied mathematics such as optimization theory, the theory of discrete-event dynamical systems, the automata theory, & the formal language theory, as well as from the allied areas of theoretical computer science & theoretical physics. The course aims at familiarizing the students with ideals, regular semirings, semimodules & building new semirings from old.

Contents

1 Hemirings & semirings
2 Definitions & examples
3 Building new semirings from old
4 Complemented elements in semrings
5 Ideals in semirings
6 Prime & semiprime ideals in semirings
7 Factor semirings
8 Morphisms of semirings
9 Regular semirings
10 Semimodules over semirings
11 Morphisms of semimodules
12 Factor semimodules
13 Free semimodules
14 Projective semimodules
15 Injective semimodules

Recommended Texts

1. Golan, J. S. (2003). *Semirings & affine equations over them: theory & applications (mathematics & its applications).* New York: Springer.
2. Glazek, K. (2002). *A guide to the literature on semirings & their applications in mathematics & information science* (1st ed.). New York: Springer.

Suggested Readings

1. Hebisch, U., & Weinert, H. J. (1998). *Semirings algebraic theory & applications in computer science* (1st ed.). Singapore: World Scientific.
2. Golan, J. S. (1999). *Semirings & their applications* (1st ed.). New York: Springer.
3. Recent research articles.

MATH -7113 Partial Differential Equations 3(3+0)

The course objectives are to learn the basics analytical methods to solve partial differential equations (PDE). Partial Differential Equations (PDEs) are at the heart of applied mathematics & many other scientific disciplines. The theory of partial differential equations (PDE) is important in pure & applied mathematics. On the one h& this is used for mathematical formulae in many phenomena from the natural sciences (electromagnetism, Maxwell's equations) or social sciences (financial markets, Black-Scholes model). On the other h& since the pioneering work on surfaces & manifolds by Gauss & Riemann partial differential equations have been at the centre of many important developments on other areas of mathematics (geometry, Poincare-conjecture). Subject of the module are four significant partial differential equations (PDEs) which feature as basic components in many applications: The transport equation, the wave equation, the heat equation, & the Laplace equation. This will discuss the qualitative behavior of solutions &, thus, be able to classify the most important partial differential equations into elliptic, parabolic, & hyperbolic type. Possible initial & boundary conditions & their impact on the solutions will be investigated. Solution techniques comprise the method of characteristics, Green's functions, & Fourier series.

Contents

1. Cauchy’s problems for linear second order equations in n-independent variables
2. Cauchy Kowalewski Theorem
3. Characteristics surfaces
4. Adjoint operations
5. Bicharacteristics Spherical & Cylindrical
6. Waves. Heatequation
7. Waveequation
8. Laplace equation
9. Maximum-Minimum Principle
10. Integral Transforms

Recommended Texts

1. Lawrence, C. E. (2010). *Partial differential equations* (2nd ed.). United States: American Mathematical Society.
2. David, L. J. (2004). *Applied partial differential equations* (2nd ed.). New York: Springer-Verlag.

Suggested Readings

1. Tynmyint, U., & Lokenath, D. (2006). *Linear partial differential equations for scientists & engineers* (4th ed.). United States: Birkhäuser Boston.
2. Jürgen, J. (2002). *Partial differential equations.* New York: Springer.

MATH-7114 Computer Aided Geometric Design 3(3+0)

The primary goals are to (1) learn about different parametric curve & surface schemes, (2) underst& the advantages & disadvantages of different geometry representations, (3) gain practical experience by implementing several CAGD techniques & user interfaces & (4) to apply CAGD methods to practical applications. Interactive graphics techniques for defining & manipulating geometrical shapes used in computer animation, car body design, aircraft design, & architectural design. In this course follow a modular approach & contribute different components to the development of an interactive curve & surface modeling system. There are two Modeling Techniques: (1) Curve Modeling Techniques have outcomes: Students will implement various curve interpolation & approximation techniques that allow the interactive specification of three-dimensional curves (e.g. Bezier, B-spline, rational curves). (2) Surface modeling techniques have outcomes: Students will implement various surface interpolation & approximation techniques that allow the interactive specification of three-dimensional surfaces (e.g. Bezier, B-spline, rational surfaces). Simple, 3D Modeling System: Students will integrate the curve & surface modules into a system that allows the user to interactively design & store simple, 3D geometries.

Contents

1. Linear interpolation, Piecewise linear interpolation blossoms, Barycentric coordinates in the plane, The de Casteljau algorithm, Properties of Bezier curves, Bernstein polynomials
2. Composite Bezier curves, Degree elevation, The variation diminishing property
3. Degree reduction, Polynomial curve constructions
4. Aitken's Algorithm, Lagrange Polynomials, Lagrange interpolation, Cubic Hermite interpolation
5. Point-normal interpolation, B-Spline curves, B-spline segments, Knot insertion, degree elevation
6. Greville Abscissae, Smoothness
7. Constructing Splines Curves, Greville interpolation, modifying B-Spline curves
8. Cubic spline interpolation, the minimum property
9. Piecewise cubic interpolation.
10. Rational Bezier & B-Spline Curves,
11. Rational Bezier curves,
12. Rational Cubic B-spline curves.

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Recommended Texts

1. Gerald, F. (2002). *Curves & surfaces for CAGD, a practical guide* (5th ed.). Massachusetts: Morgan Kaufmann Publishers.
2. Bartels, R. H., Bealty, J. C., & Beatty, J. C. (2006). *An Introduction to spline for use in computer graphics & geometric modeling.* Massachusetts: Morgan Kaufmann Publishers.

Suggested Readings

1. Josef, H., & Dieter L. (1993). *Fundamentals of computer aided geometric design.* Massachusetts: A K Peter, Ltd.
2. De Boor, C. (2001). *A practical guide to splines.* New York: Springer Verlag.
3. Wang, R. H. (2005). *Multivariate spline functions & their applications (mathematics & its applications).* Netherland: Science Press/ Kluwer Academic Publishers.

MATH -7115 **Magnetohydrodynamics-I** **3(3+0)**

Magnetohydrodynamics (MHD); also magneto-fluid dynamics or hydromagnetics is the study of the magnetic properties & behavior of electrically conducting fluids. Examples of such magneto-fluids include plasmas, liquid metals, salt water, & electrolytes. This requires an acquaintance with several areas of theoretical physics including electromagnetism, MHD pumps, biological physics, ultrasonic fields & fluid mechanics. The purpose of this subject is to explain the fundamental principles of Magnetohydrodynamics (MHD). The basic derivations of model equations representing the MHD fundamental principles are also the main focus of this course as well along with the classifications. Specifically it contains the impact on the particle motions under MHD impact for the momentum generation towards the field equations. Index notations for the MHD are explained in Euclidean Tensors & Gaussian. Stability of the MHD terms in the field equations will also be considered in the scaling analysis or basic parameters to represent in a non-dimensional quantity.

Contents

1. Basic Equations: Equations of electrodynamics
2. Equations of fluid dynamics, Ohm's law equations of magnetohydrodynamics
3. Motion of an Incompressible Fluid
4. Motion of a viscous electrically conducting fluid with linear current flow, steady state motion along a Magnetic field, wave motion of an ideal fluid
5. Small amplitude MHD Waves: Magneto-sonic waves, Alfve's waves
6. Damping & excitation of MHD waves
7. Characteristics lines & surfaces
8. Simples Waves & Shock Waves in Magnetohydrodynamics
9. Kinds of simple waves, distortion of the profile of a simple wave, discontinuities, simple ad shock waves in Relativistic magnetohydrodynamics
10. Stability & strucure of shock waves
11. Discontinuities in various quantities
12. Piston problem, oblique shock waves

Recommended Texts

1. Hans, J. P., & Stefaan, P. (2004). *Principles of magnetohydrodynamics: with applications to laboratory & astrophysical plasmas*. Cambridge: Cambridge University Press.
2. Davidson, P. A. (2016). *Introduction to magnetohydrodynamics* (2nd ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. George, W., & Arthur, S. (2006). *Engineering magnetohydrodynamics*. New York: Mac-Graw Hills.
2. Khiezer, A. I. (1975). *Plasma electrodynamics*. Oxford: Pergamon Press.
3. Anderson, J. E. (1975). *Magnetohydrodynamics shock waves*. Cambridge: M.I.T Press.

MATH -7116 **Electrodynamics-I** **3(3+0)**

Electrodynamics contains the basic concept of ionized based particles in the classical motions. This may refer to Electrohydrodynamics (EHD), also known as electro-fluid-dynamics (EFD) or electro-kinetics, is the study of the dynamics of electrically charged fluids. It is the study of the motions of ionized particles or molecules & their interactions with electric fields & the surrounding fluid. EHD printing technology, one of the printing technologies based on EHD theory can offer high resolution patterning & ejection of highly viscous ink achieve thick patterns, unlike conventional inkjet printing systems, which are thermal bubble & piezoelectric actuators. This is a core first-year graduate class whose main objectives are, to lay down the foundations of the underst&ing of the field theory, including development of important math skills in applications of tensor algebra, partial differential equations, vector calculus, etc., essential for physicists of all later specializations. The basic principles of electrohydrodynamics (EHD) will be reviewed, including governing equations & boundary conditions, the applicability of EHD models, & averaged equations in alternating external fields. Dimensionless criteria for key aspects of electric & EHD processes will be given for the stability concepts. Theoretical treatments of basic EHD phenomena, such as transient processes, sound waves in an electric field, EHD instabilities, EHD flows, & EHD heat exchange, are main focus of this course for the better underst&ings of the EHD principles.

Contents

1. Maxwell's equations
2. Electromagnetic wave equation, boundary conditions
3. Waves in conducting & non-conducting media reflection & polarization
4. Energy density & energy flux
5. Lorentz formula
6. Wave guides & cavity resonators
7. Spherical & cylindrical waves
8. Inhomogeneous wave equation
9. Retarded potentials
10. Lenard-Wiechart potentials
11. Field of uniformly moving point charge

Recommended Texts

1. Hehl, F. W., & Yuri, N. O. (2003). *Foundation of classical electrodynamics*. United States: Birkhauser.
2. Fulvio, M. (2001). *Electrodynamics*. Chicago: University of Chicago Press.

Suggested Readings

1. John, R., Reitz, F. J. M., & Robert, W. C. (2008). *Foundations of electromagnetic theory* (4^{th} ed.). United States: Addison Wesley.
2. Khiezer, A. I. (1975). *Plasma electrodynamics*. Oxford: Pergamon Press.

MATH -7117 Advanced Fluid Mechanics 3(3+0)

The main purpose of this course is not so much to feed students with "advanced" material (the topics covered do not in fact appear terribly advanced). It is instead designed to help students develop a mastery of the underlying principles & the ability to solve, quickly & efficiently, a variety of real fluid mechanics problems from first principles. The lectures present & illustrate the fundamental laws & the methods & modeling approximations that form the basis of fluid mechanics. The problems & tutorials help the students gain mastery of the material & to develop, by practice & trial & error, the mindset of an effective problem solver in fluid mechanics. This course is a survey of principal concepts & methods of fluid dynamics. Topics include mass conservation, momentum, & energy equations for continua; Navier-Stokes equation for viscous flows; similarity & dimensional analysis; lubrication theory; boundary layers & separation; circulation & vorticity theorems; potential flow; introduction to turbulence; lift & drag; surface tension & surface tension driven flows.

Contents

1. Navier-Stoke’s equation & exact solutions
2. Dimensional Analysis
3. Dynamical similarity & Reynold’s number
4. Laminar flat plate boundary layer: exact solution, momentum, integral equation, use of momentum integral Equation for flow with zero pressure gradient
5. Turbulent flow
6. Boundary layer concept & governing equations
7. Pressure gradient in boundary-layer flow
8. Reynold’s equations of turbulent motion
9. MHD equations
10. Fluid drifts
11. Stability & equilibrium problems

Recommended Texts

1. Rahman, M., & Brebbia, C. A. (2008). *Advances in fluid mechanics VII*. England: WIT Press.
2. Batchelor, G. K. (2000). *An introduction to fluid dynamics*. Cambridge: Cambridge University Press.

Suggested Readings

1. Francis, F. C. (2010). *Introduction to plasma physics & controlled fusion*. New York: springer.
2. Krall, N. A., & Trivelpiece, A. W. (1986). *Principles of plasma physics*. San Francisco: San Francisco Press, Incorporated.

MATH -7118 Advanced Analytical Dynamics-I 3(3+0)

Dynamics is traditionally defined as the classical study of motion with respect to the physical causes of motion, that is, forces & moments. Kinematics, on the other h&, is concerned with the study of motion without respect to the underlying physical causes. In this sense, kinematics is really a fundamental prerequisite upon which dynamics is constructed. For the purposes of this text, the terms dynamics & mechanics are taken to be synonymous. The choice of which term is used is based more on the academic community than on a strict technical distinction. The engineering community typically adopts the term dynamics & the physics & applied mathematics communities typically adopt the term mechanics. Analytical dynamics, or more briefly dynamics, is concerned about the relationship between motion of bodies & its causes, namely the forces acting on the bodies & the properties of the bodies, particularly mass & moment of inertia. The term dynamics is predominantly used in this text.

Contents

1. Equations of dynamic & its various forms
2. Equations of Langrange
3. Equations of Euler
4. Jacobi's elliptic functions & the qualitative & quantitative solutions of the problem of Euler & Poisson
5. The problems of Langrange & Poisson
6. Dynamical system
7. Equations of Hamilton & Appell
8. Hamilton-Jacobi theorem
9. Separable systems
10. Holder's variational principle & its consequences

Recommended Texts

1. Edmund, T. W. (2010). *A treatise on the analytical dynamics of particles & rigid bodies: With an introduction to the problem of three bodies*. London: FQ Books.
2. Bhat, R. B., & Lopez-Gomez, A. (2001). *Advanced dynamics* (1st Ed.). New Dehli: Narosa.

Suggested Readings

1. Pars, L. A. (1981). *A treatise on analytical dynamics*. Oxfords: Bow Pub.
2. Rahman, M., & Brebbia, C. A. (2008). *Advances in fluid mechanics VII*. England: WIT Press.

MATH -7119 **Elastodynamics-I** **3(3+0)**

Scope of the course is to introduce students to the theories & methods of modern earthquake geotechnical engineering. The first part of the course is devoted to illustrate fundamental notions of seismology on the origin of earthquakes & on measurement of their size through the concepts of macroseismic intensity & magnitude. Basic notions of seismometery will then be introduced together with the definition of ground motion parameters including the concept of response spectrum. The course will then proceed with the study of seismic hazard at a single site or at an extended territory & on the definition of the design earthquake using both the probabilistic & the deterministic approach. The last part of the course is dedicated to the illustration of basic notions of elastodynamics & seismic wave propagation in a continuum. These concepts will be applied to the study of local site response & of some well-known phenomena of seismic geotechnical risk such as co-seismic instability of natural slopes, cyclic mobility & liquefaction. The techniques will be developed through the boundary element research using the direct time stepping approach or the integral transform approach advanced the theory of elastic waves in solids, their time dependent behavior, & applications, especially in the area of soil-structure interaction & earthquake engineering.

Contents

1. Tensor Analysis
2. Cartesian tensors, Orthogonal rotation of axes
3. Transformation equations
4. Translation & rotation
5. Different orders of tensors. Algebra of tensors
6. contraction of tensors
7. Inner & outer multiplication of tensors
8. Symmetric & anti-symmetric tensors. Different types of tensors
9. Tensor Calculus. Differentiation & integration of tensors, application to vector analysis
10. Integral theorems in tensor form
11. Deviators, types of solid Material, Stress vector & stress tensor,
12. Analysis of strain, displacement vector
13. Lagrangian strain tensor, Physical interpretation of strain components.
14. Basic equation of theory of Elasticity
15. Generalized Hooke’s law. Types of bodies
16. Physical interpretation of Lame’s constants , Navier’s equation.

Recommended Texts

1. Shah, N. A. (2005). *Vector & tensor analysis*. Lahore: A-One publisher.
2. Zaman, F. D. (1987). *An introduction to elastodynamic*, Islamabad: National Academy of Higher Education

Suggested Readings

1. Graff, K. F. (1991). *Wave motion in elastic solids*. New York: Dover Publication Inc.
2. Rahman, M., & Brebbia, C. A. (2008). *Advances in fluid mechanics VII*. England: WIT Press.

MATH-7120 **General Relativity** **3 (3+0)**

The students shall master calculating with tensors & differential forms. They shall also be able to describe physical phenomena in different coordinate systems & to transform from one coordinate system to another. They shall be familiar with covariant derivative & covariant Lagrangian dynamics, geodesic curves, & be able to calculate the components of the Riemann curvature tensor from a given line element. They shall also be able to solve Einstein's field equations for static spherically symmetric problems & for isotropic & homogeneous cosmological models. They shall master calculating the relativistic frequency shifts for sources moving in a gravitational field, as well as the bending of light passing a spherical mass distribution. The students shall also be able to give a mathematical description of gravitational waves, as well as cosmological models in the context of general relativity.

Contents

1. Review of special relativity, tensors & field theory
2. The principles on which general relativity is based
3. Einstein's field equations obtained from geodesic deviation
4. Vacunm equation
5. The Schwarzschild exterior solution
6. Solution of the Einstein-Maxwall field equations
7. The Schwarzschild interior solution
8. The Kerr-Newmann solution (without derivation)
9. Foliations relativistic corrections to Newtonian gravity
10. Black holes, the Kruskal & Penrose diagrams
11. The field theoretic derivation of Einstein's equations
12. Weak field approximations & gravitational waves
13. Kaluza-Klein theory, isometrics. conformal transformations
14. Problems of "quantum gravity"

Recommended Texts

1. James, J. C. (2011). *The geometry of spacetime: An introduction to special & general relativity*. New York: Springer.
2. Abhay, A. (2006). *100 Years of Relativity: Space-time structure einstein & beyond*. Singapore: World Scientific Pub Co Inc.

Suggested Readings

1. Qadir, A. (1990). *Relativity: An introduction to the special theory*. Singapore: World Scientific.
2. Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1974). *Gravitation*. New York: W.H. Freeman.
3. Hawking, S. W., & Ellis, G. F. R. (1972). *The large scale structure of spacetime*. Massachusetts: Academic Press.

MATH-7121 Numerical Solutions of Ordinary Differential Equation 3 (3+0)

The main focus of this course is to get underst& for the numerical solutions of the formulated problems based upon ordinary derivatives. In mathematics, an ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable & the derivatives of those functions. The term ordinary is used in contrast with the term partial differential equation which may be with respect to more than one independent variable. Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals. *Numerical Methods* for *Ordinary Differential Equations* is a self-contained introduction to a fundamental field of *numerical* analysis & scientific computation. Many differential equations cannot be solved using symbolic computation. For practical purposes, however, such as in engineering, a numeric approximation to the solution is often sufficient. The algorithms studied here can be used to compute such an approximation. An alternative method is to use techniques from calculus to obtain a series expansion of the solution. Ordinary differential equations occur in many scientific disciplines in physics, chemistry, biology, & economics. In addition, some methods in numerical partial differential equations convert the partial differential equation into an ordinary differential equation, which must then be solved. The course also provides h&s-on experience on implementing numerical algorithms for solving engineering problems using Mathematica & MATLAB software.

Contents

1. Theory & implementation of numerical methods for initial & boundary value problems in ordinary differential equations
2. One-step, linear multi-step, Runge-Kutta of two, Runge-Kutta of three, Runge-Kutta of four
3. Extrapolation methods, Convergence, stability
4. Error estimates, Practical implementation, Study & analysis of shooting
5. Finite difference & projection methods for boundary value problems for ordinary differential equation

Recommended Texts

1. Donald, G. (2008). *Numerical solution of ordinary differential equations*. Germany: Wiley-VCH
2. Butcher, J. C. (2016). *Numerical methods for ordinary differential equations* (3rd ed.). United Kingdom: Wiley

Suggested Readings

1. Lawrence, F. S. (1994). *Numerical solution of ordinary differential equation.* New York: Chapman & Hall Mathematics CRC Press
2. Shampine, L. F., Gladwell, I., & Thompson, S. (2003). *Solving ODEs with MATLAB.* Cambridge: Cambridge University Press
3. Kendall, A., Weimin, H., & David, E. S. (2009). *Numerical solution of ordinary differential equations.* New Jersey: Wiley
4. Recent published paper

MATH-7122 **Advanced Heat Transfer** **3 (3+0)**

The aim of the course is to provide the fundamental theory for the analysis of heat transfer processes occurring in boilers, condensers, cooling towers & furnaces. The course comprises two parts: (1) Multiphase Heat Transfer, & (2) Radiative Heat Transfer. This course is intended as a one semester course for first year graduate students on convection heat transfer. Topics to be covered include basic concepts in heat transfer, differential formulation of the continuity, momentum & energy equations, exact solution of one-dimensional flow problems, boundary layer flow, approximate solutions using the integral method, heat transfer in channel flow, correlation equations in forced & free convection, flow through porous media, convection in microchannels. The study of boundary layer concept, the governing equations, simplification of momentum & energy equations will be exercise. Solutions of external flow: flow over a flat plate with constant temperature & constant heat flux conditions. Blausius solution, Pohlhausen's solution. Laminar boundary layer flow over semi-infinite flat plate: variable surface temperature. Laminar boundary layer flow over a wedge: uniform surface temperature.

Contents

1. Thermodynamics systems, Work & heat, first law of thermodynamics applied to closed & open systems, Properties of vapours in ideal gasses
2. Second law of thermodynamics & the concept of entropy
3. The external problem with reference to the vertical flat plate & horizontal circular cylinder for isothermal surface condition & constant hear flux
4. Limiting velocity & thermal fields for small & large Pr&tl numbers
5. Exact solutions for free convection from a point or line source of heat
6. The turbulent plume, The internal problem with reference to flow in Cavities
7. Lighthill’s thermosyphon, The cooling of a turbine blade.Batchelor’s work on double glazing
8. Analytical solutions of ostrich on fully developed combined free & forced convection in vertical tubes, The effects of viscous dissipation
9. Effects of non-uniform convection on forced flow in a uniformly heated horizontal circular tube following work by Monton
10. Some simple unsteady free convection problems with analytical solution

Recommended Texts

1. Holman, J. P. (2010). *Heat transfer. (*10th Ed.). New York: McGraw-Hill
2. Kays, W. M., & Crowfard, M. E. (2005). *Convective heat & mass transfer* (4th Ed.). New York: McGraw-Hill.

Suggested Readings

1. Incropera, F. P., & Dewitt, D. P. (2006). *Fundamentals of heat & mass transfer.* New York: John Wiley & Sons.
2. Sadik, K., & Yama, Y. (2013). *Convective heat transfer* (3rd Ed.). Florida*:* CRC Press .
3. Recent research articles.

MATH-7123 Introduction to Subdivision Scheme 3 (3+0)

Promote subdivision that is functional & enhances the knowledge regarding the construction of freeform curves & surfaces. In recent years, subdivision schemes have become an integral part of computer graphics in view of their extensive variety of applications in the field of visualizations, animation & image processing. Subdivision scheme deals with algorithms for free-form curves, surfaces & volumes. It plays a significant role in integrating computers & industry. Smooth curves & surfaces have pivotal importance in the field of air craft manufacturing, movie animation, computer game character design & general product design. Solutions of external flow: flow over a flat plate with constant temperature & constant heat flux conditions. Blausius solution, Pohlhausen's solution. Laminar boundary layer flow over semi-infinite flat plate: variable surface temperature. Laminar boundary layer flow over a wedge: uniform surface temperature.

Contents

1 Bilinear interpolation, The direct de Casteljau Algorithm
2 The tensor product approach & its properties
3 Bernstein polynomial, degree elevation, constructing polynomial patches: Ruled surfaces
4 Coons patches, translational surfaces, tensor product interpolation, bicubic Hermite patches
5 Composite Surfaces
6 tensor product B-spline surfaces, Matrix representation
7 Cubic spline interpolation
8 Rational Bezier & B-spline surfaces, surface of revolution
9 COONS & trimmed surface
10 Lofted Surfaces
11 Bezier Triangles: The de Casteljau algorithm, triangular blossoms
12 Bernstein polynomial, derivatives, subdivision, differentiability
13 Nonparametric patches
14 S-Patches. Surfaces with Arbitrary Topology
15 Recursive subdivision curve
16 Properties of Subdivision scheme
17 Types of Subdivision scheme

Recommended Texts

1. Gerald, F. (2002). *Curves & surfaces for CAGD. A practical guide*. (5^{th} Ed.), Burlington: Morgan Kaufmann Publishers.
2. Josef, H., & Dieter, L. (1993). *Fundamentals of computer aided geometric design.* Natick: A. K Peter Ltd.

Suggested Readings

1. Gerald, F., Josef, H., & Myung, S. (2002). *Handbook of computer aided geometric design*. Netherlands: Elsevier Science.
2. Armin, I., Ewald, Q., & Michael, S. F. (2002). *Tutorials on multiresolution in geometric modeling*. *Summer School Lecture Notes*. New York: Springer-Berlin.

MATH-7124 Nilpotent & Solvable Groups 3 (3+0)

Abelian groups are some of the easiest to underst& & most frequently met groups. They have as natural generalizations nilpotent, polycyclic & solvable groups, which generalizations are used in many different areas of algebra, geometry & topology. The course begins with the series of groups & the classification of (infinite) finitely generated Abelian groups. It continues with some of the main properties & results on nilpotent & solvable groups, illustrated with many examples, especially of linear groups. After introducing the growth function for infinite finitely generated groups & proving that nilpotent groups have polynomial growth, the course ends with the Milnor-Wolf Theorem. The latter theorem formulates a striking dichotomy in the class of solvable groups: such groups are either virtually nilpotent or of exponential growth, Students will learn fundamentals about the various subclasses of solvable groups, about the algebraic & geometric features that they have in common, as well as about those that help to differentiate them.

Contents

1. Normal & subnormal series
2. Abelian & central series, direct products
3. finitely generated Abelian groups, splitting theorems
4. Solvable groups
5. Nilpotent groups
6. Commutators subgroup, derived series, the lower & upper central series
7. Characterzation of finite Nilpotent groups
8. Fitting subgroup
9. Frattini subgroup
10. Dedekind groups
11. Supersolvable groups, solvable groups with minimal condition
12. Subnormal subgroups
13. Minimal condition on subnormal subgroups
14. The subnormal socle, the Wiel&t subgroup & Wiel&t series
15. T-groups,
16. Power automorphisms
17. Structure & Construction of finite soluble T-Groups

Recommended Texts

1. Kurzweil, H., & Stellmacher, B. (2004). *The theory of finite groups* (1st ed.). New York: Springer-Verlag.
2. Roman, S. (2011). *Fundamentals of group theory: An advanced approach* (1st ed.). New York: Springer.

Suggested Readings

1. Robinson, D. (1996). *A course in the theory of groups* (2nd ed.). New York: Springer-Verlag.
2. Doerk, K., & Hawkes, T. (1992). *Finite soluble* (1st ed.). Berlin: De Gruyter.
3. Recent research papers.

MATH-7125 **Convex Analysis** **3 (3+0)**

Convex optimization problems are far more general than linear programming problems, but they share the desirable properties of LP problems: They can be solved quickly & reliably up to very large size hundreds of thous&s of variables & constraints. The issue has been that, unless your objective & constraints were linear, it was difficult to determine whether they are convex. Convexity has been increasingly important in recent years in the study of extremum problems in many areas of applied mathematics. The purpose of this course is to provide an exposition of the theory of convex sets & functions in which applications to extremum problems play the central role. Systems of inequalities, the minimum or maximum of a convex function over a convex set, Lagrange multipliers, & minimax theorems are applications of convex analysis, along with the basic results about the structure of convex sets & the continuity & differentiability of convex functions & saddle-functions. A generalization of linear algebra is developed in which "convex bifunctions" are the analogues of linear transformations, & "inner products" of convex sets & functions are denned in terms of the extremal values in Fenchel's Duality Theorem. Each convex bifunction is associated with a generalized convex program, & an adjoint operation for bifunctions that leads to a theory of dual programs is introduced. The classical correspondence between linear transformations & bilinear functionals is extended to a correspondence between convex bifunctions & saddle-functions, & this is useful as the main tool in the analysis of saddle-functions & minimax problems.

Contents

1. Convex functions on the real line
2. Continuity & differentiability of convex functions
3. Characterizations, Differences of convex functions
4. Conjugate convex functions
5. Convex sets & affine sets
6. Convex functions on a normed linear space
7. Continuity of convex functions on normed linear space
8. Differentiable convex function on normed linear space
9. The support of convex functions
10. Differentiability of convex function on normed linear space

Recommended Texts

1. Niculescu, C. P., & Persson, L. E. (2018). *Convex functions & their applications, A contemporary approach* (2nd ed.). Canada: CMS Books in Mathematics.
2. Borwein, J. M., & Lewis, A. S. (2010). *Convex analysis & nonlinear optimization: Theory & examples* (2nd Ed.). New York: Springer.

Suggested Readings

1. Hiriart-Urruty, J. B., & Lemaréchal, C. (2004). *Fundamentals of convex analysis*. New York: Springer.
2. Roberts, A. W., & Varberg, D. E. (1973). *Convex functions*. New York: Academic Press.

MATH -7126 **Representation Theory-II** **3 (3+0)**

This course is continuation of the course Representation theory-I. The main topic of the course will be representation theory. The basic theory of linear representations of groups, with a particular focus on finite groups & representations defined over the complex numbers will be studied. We will also introduce the theory of characters as a tool for studying representations & we will develop techniques for constructing characters & character tables. We will also describe some important applications of the theory, including Burnside's famous theorem on the solubility of finite groups. We shall study the finite-dimensional representations of compact Lie groups from various points of view. The main objective will be the Weyl character formula. Then we'll turn to noncompact groups & infinite-dimensional representations. This is considerably more complicated, & at several places we shall limit ourselves to illustrating theorems in examples rather than proving them in general. This course also includes introduction to the Plancherel formula for complex groups & to the Langl&s classification.

Contents

1. Character tables & orthogonality relations
2. Normal subgroups & lifted characters
3. Some elementary character tables
4. Tensor products
5. Restriction to a subgroup, induced modules & characters
6. Algebraic integers
7. Real representations
8. Summary of properties of character tables Characters of groups of order pq
9. Characters of some p-groups
10. Character tables of the sample group of order 168, Character table of GL (2,q)
11. Permutations & characters

Recommended Texts

1. James, G., & Liebech, M. (2001). *Representations & characters of groups* (2nd ed.). Cambridge: Cambridge University Press.
2. Tullio, C. S., Fabio, S., & Filipoo, T. (2008). *Harmonic analysis on finite groups: Representation theory, gelfand pairs & markov chains* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Charles, W. C., & Irving, R. (2006). *Representation theory of finite groups & associative algebras* (1st ed.). Rhode Island: American Mathematical Society.
2. William, F., & Young, T. (2012). *Young tableaux: With applications to representation theory & geometry* (3rd ed.). Cambridge: Cambridge University Press.

MATH -7127 **Advanced Ring Theory-II** **3 (3+0)**

This course is a continuation of the course Advance Ring Theory-I. A ring is an important fundamental concept in algebra & includes integers, polynomials & matrices as some of the basic examples. Ring theory has applications in number theory & geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. In this module we shall develop ring & module theory leading to the fundamental theorems of Wedderburn & some of its applications. It develops the fundamental ideas of ring theory & is aimed at developing the student's ability to Radical, simple & semisimple artinian rings, the Artin-Wedderburn theorem & he concept of central simple algebras, the theorems of Wedderburn & Frobenius. Ring theory is powerful in terms of its scope & generality, but it can be simply described as the study of systems in which addition & multiplication are possible. A ring is an important fundamental concept in algebra & includes integers, polynomials & matrices. Ring theory has applications in number theory & geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. In this module we shall develop ring & module theory leading to the fundamental theorems of Wedderburn & some of its applications.

Contents

1. Minimal left ideals
2. Wedderburn-Artin structure theorem
3. The Brown-McCoy radical
4. the Jacobson radical
5. Connections among radical classes
6. Homomorphically closed semisimple classes

Recommended Texts

1. Huynh, D. V. (2010). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser.
2. López, P., Sergio, R., & Van, H. (2010). *Advances in ring theory.* Switzerland: Birkhäuser.

Suggested Readings

1. Wiegandt, R. (1974). *Radical & semisimple classes of rings* (1st ed.). Ontario: Queen's University.
2. Jain, S. K., & Tariq, R. S. (1997). *Advances in ring theory* (1st ed.). Basel: Birkhäuser.
3. Golan, J. S. (1999). *Semirings & their applications*. Netherlands: Springer

MATH -7128 **Theory of Group Graphs** **3 (3+0)**

Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among other fields, to solve problems that can be modelled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including group theory. The purpose of this course is two-fold: we will formally study fundamental concepts in graph theory such as flows & connectivity, the concept of matrices in graphs like Incidence matrix, Adjacency matrix, Cycle matrix & then we will examine some interactions between graphs & groups. Some problems concerning groups are best attacked by using graphs. Although in this context a graph is hardly more than a visual or computational aid, its use does make the presentation clearer & the problems more manageable. The methods are useful both in theory & in practice: they help us to prove general results about groups & results about individual groups. The course aims at familiarizing the students with graphs of group actions, graphical representations of mobius, orthogonal Affine & Euclidean groups.

Contents

1. Graphs
2. Graphs of group actions
3. Projective special linear group
4. Its action on real
5. Rational fields
6. Irrational fields
7. Graphical representations of mobius
8. Orthogonal Affine
9. Euclidean groups

Recommended Texts

1. Magnus, W., Karrass, A., & Solitar, D. (2004). *Combinatorial group theory: Presentations of groups in terms of generators & relations* (Revised ed.). New York: Dover Publications.
2. Bollobás, B. (1979). *Graphs & groups. In: graph theory. Graduate texts in mathematics*, vol 63. New York: Springer

Suggested Readings

1. Coxeter, H. S. M., & Moser, W. O. J. (1980). *Generators & relations for discrete groups* (4^{th} ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Bondy, J. A., & Murty, U. S. R. (2008). *Graph theory, Graduate texts in mathematics* (1st ed.). New York: Springer-Verlag
3. Recent research articles.

MATH -7129 **Advanced Complex Analysis-II** **3(3+0)**

This course is design to underst& the modulus of a Complex valued function & results regarding that. To Understand & develop manipulation skills in the use of Rouche’s theorem. To Underst& certain theorems like Inverse Function theorem, Hardmards three circle theorem. To underst& & learn to use Argument Principle. To underst& the principal of Analytic Continuation & the concerned results. To study the functions with positive real part. To underst& Gamma & Zeta functions, their properties & relationships. To underst& the Harmonic functions on a disc & concerned results. To underst& the factorization of entire functions having infinite zeros. To underst& range of analytic functions & concerned results. To underst& univalent functions.

Contents

1. Holomorphic functions: Review of 1-variable theory
2. Real & complex differentiability
3. Power series
4. Complex differentiable Functions
5. Cauchy integral formula for a polydise
6. Chauchy inequalities
7. The maximum principle
8. Extension of analytic functions: Hartogs figures
9. Hartogs theorem, Domains of holomorphy
10. Holomorphic convexity, theorem of Cartan Thullen
11. Levi-convexity: The Levi form, Geometric interpretation of its signature
12. E.E. Levi’s theorem, Connections with Kahlerian geometry
13. Elementary properties of plurisubharmonic functions
14. Introduction to Cohomology: Definition & examples of complex manifolds.
15. The d.operators,
16. The Poincare Lemma & the Dolbeaut Lemma,
17. The Cousin problems, introduction to Sheaf theory.

Recommended Texts

1. Kunihiko, K. (2007). *Complex Analysis, Cambridge studies in advanced mathematics*, Cambridge: Cambridge University Press.
2. Klaus, F., & Hans, G. G. (2010). *From holomorphic functions to complex manifolds*, New York: Springer.

Suggested Readings

1. Field, M. (1982). *Several complex variables & complex manifolds*, Cambridge: Cambridge University Press.
2. Grauert, H., & Fritsche, K. (1976). *Several complex variables*, New York: Springer Verlag.

MATH-7130 **Variational Inequalities** **3 (3+0)**

Variational inequality theory is a powerful unifying methodology for the study of equilibrium problems. 1 Variational inequality theory was introduced by Hartman & Stampacchia (1966) as a tool for the study of partial differential equations with applications principally drawn from mechanics. Such variational inequalities were infinite-dimensional rather than finite-dimensional as we will be studying here. The breakthrough in finite-dimensional theory occurred in 1980 when Dafermos recognized that the traffic network equilibrium conditions as stated by Smith (1979) had a structure of a variational inequality. The variational method is a powerful tool to investigate states & processes in technical devices, nature, living organisms, systems, & economics. The power of the variational method consists in the fact that many of its statements are physical or natural laws themselves. The essence of the variational approach for the solution of problems relating to the determination of the real state of systems or processes consists in the comparison of close states. The selection criteria for the actual states must be such that all the equations & conditions of the mathematical model are satisfied. The first variational theory was the Lagrange theory created to investigate the equilibrium of finite-dimensional mechanical systems under holonomic bilateral constraints (bonds).

Contents

1. Variational problems
2. Existence results for the general implicit variational problems
3. Implicit Ky Fan's inequality for monotone functions
4. Hartman-Stampacchia theorem for monotone for compact operators
5. Selection of fixed points by monotone functions
6. Variational & quasi variational inequalities for monotone operators

Recommended Texts

1. Goh, C. J. (2000). *Duality in optimization & variational inequalities* (1st Ed.). Florida: Taylor & Francis.
2. Baiocchi, C., & Capelo, A. (1984). *Variational & quasi-variational inequalities.* New York: Wiley.

Suggested Readings

1. Mosco, V. (1976). *Implicit Variational problems & quasi variational inequalities, Lecture Notes in Mathematics*-543. New York: Springer-Verlage.
2. Kravchuk, A. S., & Neittaanmaki, P. J. (2007). *Variational & quasi-variational inequalities in mechanics*. New York: Springer-Verlage, Berlin.

MATH -7131 **Field Extensions & Galois Theory** **3(3+0)**

This course is an introduction to the theory of field extensions & Galois theory. The purpose of Galois theory is to study polynomials at a deep level by using symmetries between the roots. This is a pervasive theme in modern mathematics, & Galois theory is traditionally where one first encounters it. Galois theory also enables us to prove (despite regular claims to the contrary) that there is no ruler & compass construction for trisecting an angle. On successful completion of the course the students should be able to, show familiarity with the concepts of ring & field, & their main algebraic properties, correctly use the terminology & underlying concepts of Galois theory in a problem-solving context; reproduce the proofs of its main theorems & apply the key ideas in similar arguments, calculate Galois groups in simple cases & to apply the group-theoretic information to deduce results about fields & polynomials.

Contents

1. Extension of fields: Elementary properties
2. Simple extensions
3. Algebraic extensions
4. Factorization of polynomials
5. Splitting fields
6. Algebraically closed fields
7. Separable extensions
8. Galois Theory: Automorphism of fields
9. Normal extensions
10. The fundamental theorem of Galois theory
11. Norms & traces
12. The primitive element theorem Lagrange's theorem
13. Nominal bases,
14. Applications Finite fields
15. Cyclotomic extension of rational number field
16. Cyclic extensions
17. Wedderburn's Theorem
18. Ruler-&-Compasses Construction, Solution by Radicals

Recommended Texts

1. Fraleigh, J. B., & Brland, N. (2020). *A first course in abstract algebra* (8^{th} ed.). London: Pearson.
2. Weintraub, S. (2009). *Galois theory* (2^{nd} ed.). New York: Springer-Verlag.

Suggested Readings

1. Rotman, J. J. (1995). *A introduction to the theory of groups* (4^{th} ed.). New York: Springer-Verlag.
2. Herstein, I. N. (1975). *Topics in algebra* (2^{nd} ed.). New York: John Wiley
3. Recent research articles.

MATH -7132 **Commutative Algebra-II** **3(3+0)**

This course is a continuation of the course Commutative algebra-I. Commutative algebra was born in the 19th century from algebraic geometry, invariant theory, & number theory. It has come to occupy a remarkably central role in modern mathematics, but in a different way. Today it is a mature field with activity on many fronts. The branch of mathematics which most of all draws upon commutative algebra for its structural integrity is algebraic geometry, algebraic number theory, & arithmetic geometry which is a field of mathematics that encompassed commutative algebra, classical algebraic geometry & algebraic number theory. The course aims at familiarizing the students with unique factorization domains, krull rings & factorial ring & finite factorization domains.

Contents

1. Unique Factorization Domains: Basics & examples
2. Guass Theorem, Quotient of a UFD, Nagata Theorem
3. Class Groups: Divisor classes, divisor class monoid, divisor class group
4. Krull Rings & Factorial Ring: Divisorial ideals
5. Divisors, Krull rings, stability properties, two classes of Krull rings
6. Divisor class groups, application of the theorem of Nagata, examples of factorial rings
7. Atomic Domains: Definition & examples, olynomial extension of Atomic domains
8. Domains Satisfying ACCP: Definition & examples
9. Polynomial extension of domains satisfying ACCP
10. Connection of domains satisfying ACCP & Atomic domains
11. Bounded Factorization Domains: Definition & examples.Length function
12. Characterization of BFD through length function.Polynomial extension of BFDs
13. Noetherian & Krull domains & BFDs, Half Factorial Domains: Class number of a Field
14. Carlitz Theorem, examples & basic results
15. Dedkind & Krull examples, inetegrabiltiy & HFD
16. On polynomial & polynomial like extensions
17. Finite Factorization Domains: Group of divisibility G(D) of a domain D
18. G(D) & FFD, Atomic idf-domain is FFD

Recommended Texts

1. Kemper, G. (2011). *A course in commutative algebra* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Rotman, J. (2009). *An introduction to homological algebra* (2nd ed.). New York: Springer-Verlag.

Suggested Readings

1. Chapman, S. T., & Glaz, S. (2000). *Non-Noetherian commutative ring theory* (1st ed.). New York: Springer.
2. Matsumura, H. (1986). *Commutative ring theory* (1st ed.). Cambridge: Cambridge University Press.

MATH -7133 **Commutative Semigroup Rings** **3(3+0)**

The purpose of this course to underst& the basic properties of commutative semigroup rings. Commutative Semigroup Rings was the first exposition of the basic properties of semigroup rings. This course concentrates on the interplay between semigroups & rings, thereby illuminating both of these important concepts in modern algebra. The course begins with the introduction to commutative rings & the basic notions of commutative semigroups. It continues with some of the main properties & results on cyclic semigroups & ordered semigroups, illustrated with many examples. After introducing the basic terminologies in semigroup ring, the course ends with the Monoid domains.

Contents

1. Commutative Rings: Definition & examples
2. Integral domains, unit, irreducible & prime elements in ring
3. Types of ideals, Quotient rings, Rings of fractions
4. Ring homomorphism, definitions & examples of Euclidean domains
5. Principle ideal domains & unique factorization domains
6. Definition & examples of DVRs, Dedkind & Krull domains
7. Commutative Semigroups, basic notions
8. Cyclic semigoups, Numerical Monoids
9. Ordered semigroups, congruences, Noetherian semigroups
10. Factorization in commutative Monoids
11. Semigroup Ring & its Distinguished Elements
12. Introduction of polynomial rings in one indeterminate
13. Structure of semigroup ring, Zero divisors
14. Nilpotent elements, idempotents, units
15. Ring theoretic properties of Monoid Domains
16. Integral dependence for domains & Monoid domains
17. Monoid domains as factorial domains, monoid domains as Krull domains
18. Divisor class group of a Krull Monoid domain

Recommended Texts

1. Chapman, S. T. (2005). *Arithmetical properties of commutative rings & monoids* (1st ed.). Florida: CRC Press.
2. Gilmer, R. (1972). *Multiplicative ideal theory* (1st ed.). New York: Marcel Dekker.

Suggested Readings

1. Matsumura, H. (1986). *Commutative ring theory* (1st ed.). Cambridge: Cambridge University Press.
2. Gilmer, R. (1984). *Commutative semigroup rings* (1st ed.). Chicago: The University of Chicago Press.

MATH -7134 **Cosmology** **3 (3+0)**

The main objectives of this subject are to offer students a general underst&ing, at an elementary level, of cosmology from the observational & theoretical perspectives & widen student's view on forefront knowledge & enhance their independent learning skills. The students shall master calculating with differential forms. They shall also be able to describe physical phenomena in different coordinate systems & to transform from one coordinate system to another. They shall be familiar with covariant derivative & covariant Lagrangian dynamics, geodesic curves, & would be able to calculate the components of the curvatures in different forms for a given line element. They shall also be able to solve different fields equations for static spherically symmetric & non symmetric problems & for isotropic, non isotropic & homogeneous cosmological models. They shall be able in calculating the relativistic frequency shifts for sources moving in a gravitational field, as well as the bending of light passing different spherical mass distributions. The students shall also be able to give a mathematical description of gravitational waves, as well as cosmological models in the context of general relativity.

Contents

1. Review of Relativity, historical background
2. Astronomy; Astrophysics Cosmology
3. The cosmological principle & its strong form
4. The Einstein & DeSitter universe models
5. Measurement of cosmic distance, The Hubble law & the Friedmann models
6. Steady state models. The hot big bang model
7. The microwave background, Discussion of significance of a start of time
8. Fundamentals of high energy physics
9. The chronology & composition of the universe
10. Non-baryonic dark matter, Problems of the st&ard model of cosmology
11. Bianchi space-time's, Mixmaster models, Inflationary cosmology
12. Further developments of inflationary models
13. Kaluza-Klein cosmologies, Review of material

Recommended Texts

1 Steven, L. W. (2008). *Cosmology*. Oxford: Oxford University Press.
2 Peter, S. (2010). *Extragalactic astronomy & cosmology: An introduction*. New York: Springer.

Suggested Readings

1 Peebles, P. J. E. (1993). *Principles of physical cosmology*. New Jersey: Princeton University Press
2 Ryan, M. P. Jr., & Shepley, L. C. (1975). *Homogeneous relativistic cosmologies*. New Jersey: Princeton University) Press.

MATH -7135 **Magetohydrodynamics-II** **3 (3+0)**

The aim of this course is to give students a solid grounding in fundamental plasma physics. Magnetic fields are routinely used in industry to heat, pump & levitate liquid metals. There is the terrestrial magnetic field that is maintained by fluid motion in the earth's core, the solar magnetic field, which generates sunspots & solar flares, & the galactic field that influences the formation of stars. This introductory text on magnetohydrodynamics (MHD) (the study of the interaction of magnetic fields & conducting fluids) is intended to serve as an introductory text for advanced undergraduates & graduate students in physics, applied mathematics & engineering. The material in the text is heavily weighted toward incompressible flows & to terrestrial (as distinct from astrophysical) applications. The final sections of the text, which outline the latest advances in the metallurgical applications of MHD, make the book of interest to professional researchers in applied mathematics, engineering & metallurgy. The word "magnetohydrodynamics" is derived from magneto- meaning magnetic field, hydro- meaning water, & dynamics meaning movement. The field of MHD was initiated by Hannes Alfvén for which he received the Nobel Prize in Physics in 1970.

Contents

1. Flow of Conducting Fluid past Magnetized Bodies
2. Flow of an ideal fluid past magnetized bodies
3. Fluid of finite electrical conductivity flow past a magnetized body
4. Dynamo Theories: Elsasser's theory
5. Bullard's theory, Earth's field Turbulent motion & dissipation
6. Vorticity analogy, Ionized Gases, Effects of molecular structure
7. Currents in a fully ionized gas, partially ionized gases
8. Interstellar fields, dissipation in hot & cool clouds

Recommended Texts

1. Hans, J. P. G., & Stefaan, P. (2004). *Principles of magnetohydrodynamics: With applications to laboratory & astrophysical plasmas*. Cambridge: Cambridge University Press
2. Davidson, P. A. (2016). *An introduction to magnetohydrodynamics* (2nd ed.). Cambridge: Cambridge University Press

Suggested Readings

1. George, W., & Arthur, S. (2006). *Engineering magnetohydrodynamics*. New York: Mac-Graw Hills.
2. Khiezer, A. I. (1975). *Plasma Electrodynamics*. Oxford: Pergamon Press.
3. Anderson, J. E. (1975). *Magnetohydrodynamics shock waves*. Cambridge: M.I.T Press.

MATH -7136 **Electrodynamics-II** **3 (3+0)**

This course is the second in a series on Electrodynamics beginning with Electrodynamics- I. It is a survey of basic electromagnetic phenomena: electrostatics; magnetostatics; electromagnetic properties of matter; time-dependent electromagnetic fields; Maxwell's equations; electromagnetic waves; emission, absorption, & scattering of radiation; & relativistic electrodynamics & mechanics. Electrodynamics is a branch of physics that deals with the effects arising from the interactions of electric currents with magnets, with other currents, or with themselves. The aim of this course is to give knowledge for students to work in the field of electromagnetism. After establishing the unifying connections between seemingly different phenomena in nature such as electromagnetic induction & optics, the basic properties of wave propagation, diffraction & interference will be underst&able to work in electromagnetism. Specified modeled problems can be solved easily using mathematical techniques for the simulating aspects. This course will be helpful for the underst&ings of multi-pole radiation with emphasis on dipole & radiation from accelerated charges for the frequency spectrum of the synchrotron radiation & the theory of radiation attenuation. Lagrange & Hamilton methods in field theory will also be easier to give specified applications in electrodynamics.

Contents

1. General angular & frequency distributions of radiation from accelerated charges
2. Thomson scattering
3. Cherenkov radiation
4. Fields & radiation of localized oscillating sources
5. Electric dipole fields & radiation
6. Magnetic dipole & electric quadruple fields, multipole fields
7. Multipole expansion o f the electromagnetic fields
8. Angular distributions sources of multipole radiation
9. Spherical wave expansion of a vector plane wave
10. Scattering of electromagnetic wave by a conducting sphere

Recommended Texts

1. Hehl, F. W., & Yuri, N. O. (2003). *Foundation of classical electrodynamics*. United States: Birkhauser.
2. Fulvio, M. (2001). *Electrodynamics.* Chicago: University of Chicago Press.

Suggested Readings

1. John, R., Reitz, F. J. M., & Robert, W. C. (2008). *Foundations of electromagnetic theory* (4^{th} ed.). United States: Addison Wesley.
2. Khiezer, A. I. (1975). *Plasma Electrodynamics*. Oxford: Pergamon Press.

MATH -7137 Mathematical Techniques for Boundary Value Problems 3 (3+0)

This course has a major focus on training analytical, logical thinking & learning fundamental methods for solving ordinary & partial differential equations. Both the knowledge about differential equations as well as the training of analytical faculties will be useful for the students in the course of their further studies. In mathematics, in the field of differential equations, a boundary value problem is a differential equation together with a set of additional constraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions. Boundary value problems arise in several branches of physics as any physical differential equation will have them. Problems involving the wave equation, such as the determination of normal modes, are often stated as boundary value problems. A large class of important boundary value problems is the Sturm–Liouville problems. The analysis of these problems involves the Eigen functions of a differential operator. To be useful in applications, a boundary value problem should be well posed.

Contents

1. Green's function method with applications to wave-propagation
2. Keller Box scheme
3. Local Non-Similarity method
4. Shooting method
5. A survey of transform techniques
6. Wiener-Hopf technique with applications to diffraction problems

Recommended Texts

1. Stakgold, I. (1987). *Boundary value problems of mathematical physics.* Philadelphia: Society for Industrial & Applied Mathematics
2. Na, T. Y. (2012). *Computational methods in engineering boundary value problems.* New York: Academic Press

Suggested Readings

1. Noble, B. (1998). *Methods based on the wiener-hopf technique for the solution of partial differential equations* (2nd ed.). New York: American Mathematical Society
2. Recent research articles

MATH -7138 **Advanced Analytical Dynamics-II** **3(3+0)**

Scope of the course is to introduce students to the theories & methods of modern earthquake geotechnical engineering. The first part of the course is devoted to illustrate fundamental notions of seismology on the origin of earthquakes & on measurement of their size through the concepts of macroseismic intensity & magnitude. Basic notions of seismometery will then be introduced together with the definition of ground motion parameters including the concept of response spectrum. The course will then proceed with the study of seismic hazard at a single site or at an extended territory & on the definition of the design earthquake using both the probabilistic & the deterministic approach. The last part of the course is dedicated to the illustration of basic notions of elastodynamics & seismic wave propagation in a continuum. The main is to Provide a unique bridge between the foundations of analytical mechanics & application to multi-body dynamical systems. To study established principles in mechanics are presented in a thorough & modern way. The course build up from general mathematical foundations, an extensive treatment of kinematics, & then to a rigorous treatment of conservation & variational principles in mechanics. Parallels will be drawn between the different approaches, providing the reader with insights that unify his or her underst&ing of analytical dynamics. Additionally, a unique treatment will be discussed on task space dynamical formulations that map traditional configuration space representations into more intuitive geometric spaces.

Contents

1. Groups of continuous transformation & Poincare's equations
2. Systems with one degree of freedom
3. Singular points
4. Cyclic characteristics of systems with a degree of freedom
5. Ergodie theorem
6. Metric indecompossability stability of motion

Recommended Texts

1. Edmund, T. W. (2010). *A treatise on the analytical dynamics of particles & rigid bodies: With an introduction to the problem of three bodies*. London: FQ Books.
2. Bhat, R. B., & Lopez-Gomez, A. (2001). *Advanced dynamics* (1st Ed.). New Dehli: Narosa.

Suggested Readings

1. Pars, L. A. (1981). *A treatise on analytical dynamics*. Oxfords: Bow Pub.
2. Khiezer, A. I. (1975). *Plasma Electrodynamics*. Oxford: Pergamon Press.

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MATH -7139 **Elastodynamics-II** **3(3+0)**

The aim of this course is to give students advance topics in elastodynamics. The development of linear elastodynamics in pure stress-based formulation began over half-a-century ago as an alternative to the classical displacement-based treatment that came into existence two centuries ago in the school of mathematical physics in France. While the latter approach – fundamentally based on the Navier displacement equation of motion – remains the conventional setting for analysis of wave propagation in elastic bodies, the stress-based formulation & the advantages it offers in elastodynamics & its various extensions remain much less known. Since the key mathematical results of that formulation, as well as a series of applications, originated with J. Ignaczak in 1959 & 1963, the key relation is named the Ignaczak equation of elastodynamics. This course presents the main ideas & results in the stress-based formulation from a common perspective, including (i) a history of early attempts to find a pure stress language of electrodynamics, (ii) a proposal to use such a language in solving the natural traction initial-boundary value problems of the theory, & (iii) various applications of the stress language to elastic wave propagation problems. Finally, various extensions of the Ignaczak equation of elastodynamics focused on dynamics of solids with interacting fields of different nature (classical or micropolar thermoelastic, fluid-saturated porous, piezoelectro-elastic) as well as nonlinear problems are included.

Contents

1. Derivation of equation of motion
2. Helmotz theorem
3. Components of displacement in terms of potentials
4. Strain components, stress components
5. Waves & vibrations in strings, Waves in long string
6. Reflection & transmission at boundaries
7. Free vibration of a finite string, Forced vibration of a string,
8. The string on an elastic base dispersion
9. Pulses in a dispersive media
10. The string on a viscous sub grade

Recommended Texts

1. Shah, N. A. (2005). *Vector & tensor analysis*. Lahore: A-One publisher.
2. Zaman, F. D. (1987). *An introduction to elastodynamic*, Islamabad: National Academy of Higher Education

Suggested Readings

1. Graff, K. F. (1991). *Wave motion in elastic solids*. New York: Dover Publication Inc.
2. Recent research articles.

MATH-7140 **Design Theory** **3(3+0)**

Design theory is a branch of combinatorics. Its traditional roots are in the design of experiments, but it has found recent applications in cryptography, coding theory & communication networks. Over the past several decades, algebra has become increasingly important in combinatorial design theory. The flow of ideas has for the most part been from algebra to design theory. Moreover, despite our successes, fundamental algebraic questions in design theory remain open. It seems that new or more sophisticated ideas & techniques will be needed to make progress on these questions. In the meantime, design theory is a fertile source of problems that are ideal for spurring the development of algorithms in the active field of computational algebra. Combinatorial designs are used to determine which patients receive which treatments in such a way that if a given response is observed, then the structure of the design would indicate the treatment that caused it. Modern applications are also found in a wide gamut of areas including; Finite geometry, tournament scheduling, lotteries, mathematical chemistry, mathematical biology, algorithm design & analysis, networking, group testing & cryptography. This Course includes an introduction to Design Theory including a selection of topics from Latin squares, Steiner triple systems, balanced incomplete block designs, graph decompositions, projective & affine designs. The course should allow students subsequently to read further in these areas, & to apply their knowledge of graph theory & design theory to other appropriate fields.

Contents

1. Basic definitions & properties
2. Related structure, The incidence matrix
3. Graphs, residual structures
4. The Bruck - Ryser-Chowla theorem
5. Singer groups & difference sets
6. Arithmetical relations & Hadamard 2- designs
7. Projective & affine planes
8. Latin squares, nets. Hadamard matrices & Hadamard 20 design
9. Biplanes, strongly regular graphs
10. Cameron’s theorem & Hadamard 3-desings
11. Steiner triple systems, The Mathieu groups

Recommended Texts

1. Beth, T., Jungnickel, D., & Lenz, H. (1999). *Design theory: Volume 1*. Cambridge: Cambridge University Press.
2. Wallis, W. D. (2016). *Introduction to combinatorial designs*. Boca Raton: CRC Press.

Suggested Readings

1. Cameron, P. J., Van Lint, J. H., & Cameron, P. J. (1991). *Designs, graphs, codes & their links: Volume 3*. Cambridge: Cambridge University Press.
2. Cameron, P. J. (1999). *Permutation groups: Volume 45*. Cambridge: Cambridge University Press
3. Lindner, C. C., & Rodger, C. A. (2017). *Design theory*. Florida: CRC press

MATH-7141 **Acoustics** **3(3+0)**

This course introduces acoustics by using the concept of impedance. The course starts with vibrations & waves, demonstrating how vibration can be envisaged as a kind of wave, mathematically & physically. They are realized by one-dimensional examples, which provide mathematically simplest but clear enough physical insights. Then the part 1 ends with explaining waves on a flat surface of discontinuity, demonstrating how propagation characteristics of waves change in space where there is a distributed impedance mismatch. Describe the characteristics of spaces designed for effective listening, working, learning, & other functions. Describe environmental acoustics in terms of acoustic enhancement & environmental noise control. Relate the scientific principles of acoustic design to the design & construction of comfortable spaces. Students study concepts and theories of acoustics, such as frequency and modes, elastic media, transmission and radiation. They also learn to measure noise and study plane, spherical and standing sound waves. Other topics may include looking at how sound is made in tubes and cavities.

Contents

1. Fundamentals of vibrations, Energy of vibration
2. Damped & free oscillations
3. Transient response of an oscillator vibrations of strings
4. Membranes & plates, forced vibrations
5. Normal modes, Acoustic waves equation & its solution
6. Equation of state, equation of cout, Euler's equations
7. Linearized wave equation, speed of sound in fluid
8. Energy density, acoustic intensity, specific acoustic impedance
9. Spherical waves, transmission
10. Transmission from one fluid to another (Normal incidence) reflection at a surface of solid (normal & oblique incidence)
11. Absorption & attenuation of sound waves in fluids
12. Pipes cavities waves guides, under water acoustics

Recommended Texts

1. Everest, F. A., & Pohlmann, K. C. (2015). *Master handbook of acoustics* (5th ed.). New York: McGraw-Hill
2. Kinsler, L. E., Frey, A. R., Coppens, A. B., & Sanders, J. V. (1999). *American: Fundamentals of acoustics*. New York: Wiley-VCH.

Suggested Readings

1. Morse, P. M., & Ingard, K. U. (1986). *Theoretical acoustics*. New Jersey: Princeton university press.
2. Redfern, F. R., & Munson, R. D. (1982). *Acoustic emission source location: a mathematical analysis* (Vol. 8692). Washington: US Department of the Interior, Bureau of Mines.
3. Vorländer, M. (2007). *Auralization: fundamentals of acoustics, modelling, simulation, algorithms & acoustic virtual reality*. New York: Springer Science & Business Media.

MATH-7142 **Combinatorics** **3(3+0)**

Combinatorics is an area of mathematics primarily concerned with counting, both as a means & an end in obtaining results, & certain properties of finite structures. It is closely related to many other areas of mathematics & has many applications ranging from logic to statistical physics, from evolutionary biology to computer science, etc. Combinatorics is a growing field utilized in data science, computer science, statistics, probability, engineering, physics, business management, & everyday life. This course is a great introduction with some specialized topics. In this course, students will become familiar with fundamental combinatorial structures that naturally appear in various other fields of mathematics & computer science. They will learn how to use these structures to represent mathematical & applied questions, & they will become comfortable with the combinatorial tools commonly used to analyze such structures. Given a hypothetical combinatorial object that must satisfy certain properties, students will learn how to prove the existence or non-existence of the object, compute the number of such objects, & underst& their underlying structure.

Contents

1. Elementary concepts of several combinatorial structures
2. Recurrence relations & generating functions
3. Principle of inclusion & exclusion
4. Latin squares & SDRs. Steiner system
5. A direct construction, A recursive construction
6. Packing & covering
7. Linear algebra over finite fields
8. Gaussian coefficients, The pigeonhole Principle
9. Some special cases, Ramsey's theorem
10. Bounds for Ramsey numbers & applications
11. Automorphism groups & permutation groups
12. Enumeration under group action

Recommended Texts

1. Lothaire, M., & Lothaire, M. (2002). *Algebraic combinatorics on words* (Vol. 90). Cambridge: Cambridge university press.
2. Stanley, R. P. (2007). *Combinatorics & commutative algebra* (Vol. 41). Cambridge: Springer Science & Business Media.

Suggested Readings

1. Brualdi, R. A. (1977). *Introductory combinatorics*. London: Pearson Education.
2. Brualdi, R. A., & Cvetkovic, D. (2008). *A combinatorial approach to matrix theory & its applications*. Florida: CRC press.
3. Flajolet, P., Lam, T. Y., & Lutwak, E. (1987). *Combinatorial geometries* (Vol. 29). Cambridge: Cambridge University Press.

MATH-7143 **Theory of Majorization** **3(3+0)**

In mathematics, majorization is a preorder on vectors of real numbers. For a vector, we denote by the vector with the same components, but sorted in descending order. Given, we say that weakly majorizes (or dominates) from below written as iff. Many problems arising in signal processing & communications involve comparing vector-valued strategies or solving optimization problems with vector- or matrix-valued variables. Majorization theory is a key tool that allows us to solve or simplify these problems. This course plays a fundamental role in nearly all branches of mathematics, inequalities are usually obtained by ad hoc methods rather than as consequences of some underlying “theory of inequalities.” For certain kinds of inequalities, the notion of majorization leads to such a theory that is sometimes extremely useful & powerful for deriving inequalities. Moreover, the derivation of an inequality by methods of majorization is often very helpful both for providing a deeper underst&ing & for suggesting natural generalizations.

Contents

1. Motivation & Basic Definitions
2. Majorization as a Partial Ordering
3. Order-Preserving Functions
4. Partial Orderings Induced by Convex Cones
5. Partial Orderings Generated by Groups of Transformations
6. Majorization for Vectors of Unequal Length
7. Majorization for Infinite Sequences
8. Majorization for Matrices, Lorenz Ordering
9. Majorization & Dilations, Complex Majorization

Recommended Texts

1. Marshall, A. W., Olkin, I., & Arnold, B. C. (1979). *Inequalities: theory of majorization & its applications*. New York: Academic press.
2. Bhatia, R. (2013). *Matrix analysis* (Vol. 169). New York: Springer Science & Business Media.

Suggested Readings

1. Peajcariaac, J. E., & Tong, Y. L. (1992). *Convex functions, partial orderings, & statistical applications*. New York: Academic Press
2. Arnold, B. C. (2012). *Majorization & the Lorenz order: A brief introduction*. New York: Springer Science & Business Media.
3. Jorswieck, E., & Boche, H. (2007). *Majorization & matrix-monotone functions in wireless communications* (Vol. 3). Netherl&: Now Publishers Inc.
4. Latest Research Papers.

MATH-7144 Inequalities involving Convex Functions 3 (3+0)

The inequalities play an important role in almost all branches of mathematics as well as in other areas of science. The basic work “Inequalities” by Hardy, Littlewood & Pólya appeared in 1934 & the books “Inequalities” by Beckenbach & Bellman published in 1961 & “Analytic Inequalities” by Mitrinovi´c published in 1970 made considerable contributions to this field & supplied motivations, ideas, techniques & applications. Since 1934 an enormous amount of effort has been devoted to the discovery of new types of inequalities & to the application of inequalities in many parts of analysis. The usefulness of mathematical inequalities is felt from the very beginning & is now widely acknowledged as one of the major driving forces behind the development of modern real analysis. The theory of inequalities is in a process of continuous development state & inequalities have become very effective & powerful tools for studying a wide range of problems in various branches of mathematics. This theory in recent years has attracted the attention of many researchers, stimulated new research directions, & influenced various aspects of mathematical analysis & applications. Among the many types of inequalities, those associated with the names of Jensen, Hadamard & Hermite have deep roots & made a great impact on various branches of mathematics.

Contents

1. Jensen’s & related inequalities
2. Some general inequalities involving convex functions
3. Hadamard’s inequalities
4. Inequalities of Hadamard type I
5. Inequalities of Hadamard type II
6. Some inequalities involving concave functions
7. Miscellaneous inequalities

Recommended Texts

1. Pachpatte, B. G. (2005). *Mathematical inequalities*. New York: Elsevier.
2. Pečarić, J., Proschan, F., & Tong, Y. C. (1992). *Convex functions, partial orderings & statistical applications* (Vol. 187). Cambridge: Academic Press, Boston, Mass.

Suggested Readings

1. Mitrinovic, D. S., Pečarić, J., & Fink, A. M. (1993). *Classical & new inequalities in analysis*. Netherl&s: Kluwer Academic Publishers.
2. Recent research articles.

MATH-7145 **Harmonic Analysis** **3 (3+0)**

Harmonic analysis is the art of decomposing functions & operators into simpler building blocks that can be analyzed separately & then reassembled together. The exponential functions provide an excellent analysis tool, Fourier analysis is the study of such decompositions. Fourier (or harmonic) analysis is a discipline which lies in the intersection of classical & functional analysis & has many applications to differential equations, operator theory, probability & statistics, number theory, & many other areas of mathematics, physics, & engineering. It plays a central role in mathematics because its main idea is to extend decomposition of a vector in an orthogonal basis of a finite dimensional space to orthogonal series expansions or analogous integral representations of functions. The course will enhance research, inquiry & analytical thinking abilities. Following topics will be studied. The course will treat central concepts and results in modern harmonic analysis, which are developments from classical Fourier analysis. One possible theme may be harmonic analysis related to the study of singular integrals and complex and real methods. Some key concepts are maximal functions, Calderon-Zygmund decompositions, the Hilbert transform, Littlewood-Paley theory, Hardy spaces, Carleson measures, Cauchy integrals, singular integral operators.

Contents

1. Topology, Sets & Topologies
2. Separation axioms & related theorems
3. The Stone- Weierstrass theorem, Cartesian products & weak topology
4. Banach spaces, Normed linear spaces
5. Bounded linear transformations, Linear functional
6. The weak topology for X*, Hilbert space
7. Involution on ß (H), Integration, The Daniell integral
8. Equivalence & measurability, The real LP –spaces
9. The conjugate space of LP
10. Integration on locally compact Hausdorff spaces
11. The complex LP –spaces, Banach Algebras
12. Definition & examples, Function algebras
13. Maximal ideals, Spectrum, adverse Banach algebras
14. Elementary theory, The maximal ideal space of a commutative Banach algebra
15. Some basic general theorems

Recommended Texts

1. Deitmar, A., & Echterhoff, S. (2014). *Principles of harmonic analysis*. New York: Springer.
2. Katznelson, Y. (2004). *An introduction to harmonic analysis*. Cambridge: Cambridge University Press.

Suggested Readings

1. Varadarajan, V. S. (1999). *An introduction to harmonic analysis on semisimple lie groups*. New York: Cambridge University Press.
2. Heil, C. (2009). *Introduction to harmonic analysis*. Boston: Birkhäuser.
3. Wang, B., Huo, Z., Guo, Z., & Hao, C. (2011). *Harmonic analysis method for nonlinear evolution equations, I*. World Scientific.

MATH-7146 **Research Methodology** **3 (3+0)**

The purpose of research is to discover answers to questions through the application of scientific procedures. The main aim of research is to find out the truth which is hidden & which has not been discovered as yet. The students will gain familiarity with a phenomenon or to achieve new insights into it (studies with this object in view are termed as exploratory or formulative research studies). The students should be able to identify the overall process of designing a research study from its inception to its report. The course will treat central concepts and results in modern harmonic analysis, which are developments from classical Fourier analysis. One possible theme may be harmonic analysis related to the study of singular integrals and complex and real methods. Some key concepts are maximal functions, Calderon-Zygmund decompositions, the Hilbert transform, Littlewood-Paley theory, Hardy spaces, Carleson measures, Cauchy integrals, singular integral operators.

Contents

1. Scientific statements, hypothesis, model
2. Theory & Law, Types of research, Problem definition
3. Objectives of the research, research design, data collection
4. Data analysis, Interpretation of results, validation of results
5. Limitation of Science, calibration, Sensitivity
6. Least count & reproducibility, Stability & objectivity
7. Difference between accuracy & precision
8. Literature search, defining problem, Feasibility study
9. Pilot projects / field trials, Formal research proposal
10. Budgeting & funding, Progress report, final technical & fiscal report
11. Purpose of experiment, good & bad experiments
12. Inefficient experiments, null & alternative hypothesis
13. Alpha & beta errors, Relationship of alpha & beta errors to sensitivity & specificity
14. Designing efficient experiments, Simple r&om sampling
15. Systematic sampling, Stratified sampling, cluster sampling
16. Convenience sampling, judgment sampling, quota sampling
17. Snow ball sampling , Identifying variables of interest & their interactions
18. Operating characteristic curves, power curves, Surveys & field trials
19. Submission of a paper, role of editor, Peer-review process
20. Importance of citations, impact factor, Plagiarism

Recommended Texts

1. Kumar, R. (2010). *Research methodology: A step-by-step guide for beginners* (3rd ed.). New York: Sage Publications Ltd.
2. Harre, R. (2002). *Great scientific experiments: Twenty experiments that changed the world*: New York: Dover Pub.

Suggested Readings

1. Day, R. A. (1979). *How to write & publish a scientific paper*. USA: ISI Press, Philadelphia.
2. Diamond, W. J. (1989). *Practical experiment designs for scientists & engineers* (2nd ed.). New York: John Wiley.

MATH-7147 **Integral Transform** **3 (3+0)**

The goals for the course are to gain a facility with using the transform, both specific techniques & general principles, & learning to recognize when, why, & how it is used. Together with a great variety, the subject also has a great coherence, & the hope is students come to appreciate both. The Students will be able to know the use of Laplace transform in system modeling, digital signal processing, process control, solving Boundary Value Problems. Students will be able to use Fourier transform in communication theory & signal analysis, image processing & filters, data processing & analysis, solving partial differential equations for problems on gravity. The students will be able to use Z-transform in the characterization of Linear Time Invariant system (LTI), in development of scientific simulation algorithms. The student will be able to use the Mellin transform to solve various problems. The students will be able to solve different problems by using the Hankel transform. In mathematics, an integral transform maps an equation from its original domain into another domain where it might be manipulated & solved much more easily than in the original domain. The solution is then mapped back to the original domain using the inverse of the integral transform.

Contents

1. Laplace transform
2. Application to integral equations
3. Fourier transforms
4. Fourier sine & cosine transform
5. Inverse transform, application to differentiation
6. Convolutions theorem, application to partial differential equations
7. Hankel transform & its applications
8. Application to integration
9. Mellin transform & its applications
10. Abel transform
11. Hilbert transform
12. Jacobi transform

Recommended Texts

1. Davies, B. (2002). *Integral transforms & their applications* (3rd ed.). New York: Springer.
2. Trigub, R. M., & Belinsky, E. S. (2010). *Fourier analysis & approximation of functions*. New York: Springer.

Suggested Readings

1. Pinkus, A., & Zafrany, S. (1997). *Fourier series & integral transforms*. Cambridge: Cambridge University Press.
2. Vasistha, R., & Gupta, R. K. (2007). *Integral transform*. India: Krisna Prakashan Media Pvt. Ltd.
3. Recent research articles.

MATH-7148 **Advanced Numerical Analysis** **3 (3+0)**

Advanced numerical analysis is essential in making numerical weather prediction feasible. Computing the trajectory of a spacecraft requires the accurate measuring in the target achievements. This course is the continuation of Numerical Analysis. The student will learn state-of-the-art algorithms for solving ordinary differential equations, nonlinear systems, & optimization problems. Moreover, the analysis of these algorithms & their efficient implementation will be discussed in some detail. The emphasis will be on both the analysis & the implementation of these methods. The same idea for the approximation of the solution by a series expansion (truncated) will be used for the numerical computations. The student will learn some basic theoretical results on approximations for the issues of stability & convergence, on practical algorithms for implementing numerical methods, & on designing efficient & accurate spectral algorithms for solving mathematical problems. More advanced techniques for numerical computations will be introduced for the better learning of MATLAB skills in numerical methods, programming & graphics. After this, one can learn easily with Scilab for advanced numerical analysis package similar to MATLAB or Octave for a complete GUI & Xcos which is alternative to Simulink frequently used in weather forecasting in super computers for advanced technologies.

Contents

1. Introduction, Euler's method
2. The improved & modified Euler's method
3. Runge-Kutta method, Milnes method
4. Hammign's methods, Initial value problem
5. The special cases when the first derivative is missing
6. Boundary value problems
7. The simultaneous algebraic equations method
8. Iterative methods for linear equations
9. Gauss-Siedel method, Relaxation methods
10. Vector & matrix norms, Sequences & series of matrices
11. Graph Theory, Directed graph of a matrix
12. Strongly connected & irreducible matrices
13. Grerschgoin theorem, Symmetric & positive definite matrices
14. Cyclic-Consistently ordered matrices
15. Choice of optimum value for relaxation parameter

Recommended Texts

1. Richard, L. B., & Faires, J. D. (2010). *Numerical analysis* (9^{th} ed.). New York: Brooks Cole.
2. Stanislaw, R. (2008). *Fundamental numerical methods for electrical engineering. Lecture notes in electrical engineering.* Berlin: Springer.

Suggested Readings

1. Gerald, C. F., & Wheatley, P. O. (1994). *Applied numerical analysis.* Boston: Addison-Wesley Publishing Company.
2. Recent research articles.

MATH-7149 **Generalized Special Functions** **3 (3+0)**

The main aim of this course is the study of the properties & relations of special functions such as incomplete gamma, beta functions, zeta, hypergeometric, confluent hypergeometric functions, Bessel functions & generalized hypergeometric functions. Furthermore, the properties, relations & applications of these special functions are also discussed in the form a new direction k>0. The students will able to find some results & applications associated with special k-functions. Special functions are those mathematical functions which more or less have established their identities due to their undoubted usefulness in mathematical analysis, functional analysis, physics, chemistry, & other fields of science, technology & industry. Many special functions appear as solutions of differential equations or integrals of elementary functions. Therefore, tables of integrals usually include descriptions of special functions, & tables of special functions include most important integrals; at least, the integral representation of special functions. Because symmetries of differential equations are essential to both physics & mathematics, the theory of special functions is closely related to the theory of Lie groups & Lie algebras, as well as certain topics in mathematical physics.

Contents

1. Infinite products
2. Properties & applications of special functions
3. The incomplete gamma, beta functions
4. Zeta functions
5. The hypergeometric functions & identities
6. Generalized hypergeometric functions
7. Bessel functions
8. The confluent hypergeometric functions
9. Introduction to *q*-series, *k*- hypergeometric functions
10. Generalized *k*-hypergeometric functions
11. Confluent *k*-hypergeometric functions

Recommended Texts

1. Richard, B. (2016). *Special functions & orthogonal polynomials*. Cambridge: Cambridge University Press.
2. Rainville, E. D. (1971). *Special functions* (3rd ed.). New York: The Macmillan Company.

Suggested Readings

1. Andrews, G. E., Richard, A., & Roy, R. (2000). *Special functions* (1st ed.). Cambridge: Cambridge University Press.
2. Mathai, A. M., & Houbold, H. J. (2008). *Special functions for applied scientists*. New York: Springer Science & Business Media, LLC.
3. Singh, U. P., & Denis, R.Y. (2001). *Special functions & their applications*. New Dehli: Dominant Publishers & Distributors.

MATH-7150 **Scientific Computation** **3(3+0)**

The main goal of this course is to learn how to solve mathematical problems that require computational power. MATLAB/Mathematica will be the preferred platform for pursuing this goal. No prior experience with MATLAB is required. The labs are designed to introduce the student to MATLAB & how it can be used in solving mathematical problems. The course is an introduction to numerical methods with emphasis on algorithms, analysis of algorithms, and computer implementation issues. We will focus on the topics such as solution of nonlinear equations; interpolation and curve fitting; numerical differentiation and integration; solving ordinary differential equation; solving linear systems by direct methods and iterative methods; matrices eigenvalues and eigenvectors. At the end of this course the student should be able to: (1) Decide when & how much to rely on a computer in the process of solving a mathematical problem. (2) Develop the necessary code, test the code & analyze the outcome of the code. (3) Use the computer output to present the solution to the original problem & gain further insight in the problem & its solution.

Contents

1. Vector & matrix operation, Building exploratory environments
2. Floating point arithmetic, Error analysis
3. The interpolating polynomial, Piecewise linear interpolation
4. Piecewise cubic Hermite interpolation
5. Different degree spline interpolations
6. Shape-preserving interpolants, Bisection method
7. Newton’s method, Second method
8. Inverse quadratic interpolation, Quasi-Newton’s method
9. Basic quadrature rules, Adaptive quadrature,
10. Least squares data fitting, Models & data curve fitting
11. Norms, The QR factorization
12. Pseudo inverse, Eigenvalues & singular values

Recommended Texts

1. Moler, D. (2004). *Numerical computing with Matlab*. Philadelphia: Society for Industrial & Applied Mathematics.
2. Lindfield, G. R., & Penny, J. E. T. (2012). *Numerical methods using Matlab (*3rd ed.). Massachusetts: Academic Press.

Suggested Readings

1. Burdern, R. L., & Faires, J. D. (2004). *Numerical analysis* (8th ed.). New York: Brooks Cole.
2. Hoffman, J. D. (1992). *Numerical methods for engineers & scientists* (2nd ed.). New York: Marcel Dekker, Inc.
3. Ramin, S. E. (2017). *Numerical methods for engineers & scientists using Matlab*. Florida: CRC Press.

MATH-7151 Mathematical Modeling-I 3(3+0)

This course is an introduction to mathematical modeling based on the use of elementary functions to describe and explore real-world phenomena and data. Linear, exponential, logarithmic, and polynomial function models are examined closely and are applied to real-world data in course assignments and projects. Other function models may also be considered. Throughout the course, computational tools (graphing calculators, spreadsheets, etc.) are used to implement, examine, and validate these models. Students are expected to actively engage in the modeling process by questioning phenomena, collecting or creating data, and using computational tools to develop their models and evaluate their efficacy. The main objective of the course is to introduce mathematical modeling, that is, the construction & analysis of mathematical models inspired by real life problems. The course will present several modeling techniques & the means to analyze the resulting systems. Students can expect to acquire the following knowledge & skills at the end of the Masters course: (1) Theory of partial derivative equations, numerical discretization, error analysis; (2) Continuous & discrete optimization, calculus of variations, game theory; (3) Finite or infinite dimensional control theory, optimal control theory, inverse problems; (4) Analysis, simulation & modeling tools that are used in life sciences; (5) Scientific computing, scientific computation, parallel computation, computer-aided design. Students will also acquire knowledge in various applied fields: computer science, biology, physics, mechanics, & economics.

Contents

1. Introduction to Modeling, Collection & interpretation of data
2. Setting up & developing models
3. Checking models. Consistency of models
4. Dimensional analysis
5. Discrete models, Multivariable models, Matrix models
6. Continuous models
7. Modeling rates of changes, Limiting models
8. Graphs of functions as models
9. Periodic models
10. Modeling with difference equations
11. Linear, Quadratic & Non-Linear Models.

Recommended Texts

1. Edwards, D., & Hamson, M. (1996). *Mathematical modeling skills*. New York: Macmillan Press Ltd.
2. Giordano, F. R., Weir, M. D., & Fox, W. P. (2003). *A first course in mathematical modeling*. New York: Brooks Cole.

Suggested Readings

1. Thomas, W., & Mark, B. (2015). *Methods of mathematical modelling: Continuous systems & differential equations*. New York: Springer.
2. Sandip, B. (2014). *Mathematical modeling: Models, analysis & applications*. Florida: CRC Press.
3. Clive L. D. (2004). *Principles of mathematical modeling* (2nd ed.). Cambridge: Academic Press.

MATH-7152 Mathematical Modeling-II 3(3+0)

The main objective of the course is to introduce mathematical modeling, that is, the construction & analysis of mathematical models inspired by real life problems. The course will present several modeling techniques & the means to analyze the resulting systems. Students can expect to acquire the following knowledge & skills at the end of the Masters course: (1) Theory of partial derivative equations, numerical discretization, error analysis; (2) Continuous & discrete optimization, calculus of variations, game theory; (3) Finite or infinite dimensional control theory, optimal control theory, inverse problems; (4) Analysis, simulation & modeling tools that are used in life sciences; (5) Scientific computing, scientific computation, parallel computation, computer-aided design. Students will also acquire knowledge in various applied fields: computer science, biology, physics, mechanics, & economics. Throughout the course, computational tools (graphing calculators, spreadsheets, etc.) are used to implement, examine, and validate these models. Students are expected to actively engage in the modeling process by questioning phenomena, collecting or creating data, and using computational tools to develop their models and evaluate their efficacy.

Contents

1. Modeling with Differential Equations
2. Exponential growth & decay
3. Linear
4. Non-linear systems of differential equations
5. Modeling with integration
6. Modeling with r&om numbers
7. Simulating qualitative r&om variables
8. Simulating discrete r&om variables
9. St&ard models
10. Monte Carlo simulation
11. Fitting models to data
12. Bilinear interpolation & Coons patch

Recommended Texts

1. Giordano, F. R., Weir, M. D., & Fox, W. P. (2003). *A first course in mathematical modeling*. New York: Brooks/Cole.
2. Dilwyn, E., & Mike, H. (2001). *Guide to mathematical modeling (mathematical guides)*. London: Palgrave Macmillan

Suggested Readings

1. Thomas, W., & Mark, B. (2015). *Methods of mathematical modeling: continuous systems & differential equations*. New York: Springer.
2. Sandip, B. (2014). *Mathematical modeling: models, analysis & applications*. Florida: Chapman & Hall, CRC Press.
3. Clive L. D. (2004). *Principles of mathematical modeling* (2nd ed.). Cambridge: Academic Press.

MATH-7153 **Computer Graphics** **3(3+0)**

Computer Graphics is the illustration field of Computer Science. Its use today spans virtually all scientific fields & is utilized for design, presentation, education & training. Computer Graphics & its derivative, visualization, have become the primary tools by which the flood of information from Computational Science is analyzed. The effective construction of three-dimensional computer-generated illustrations is not only a computer science problem; it is a problem that involves other fields as well. For example, it depends heavily on mathematics for its geometric basis & computational algorithms, & it depends on Physics for its principles of geometric optics (the reflection of light from surfaces determines the displayed color of the surface). It is also the study of a field that provides methods by which illustrations can be generated by others. Basic principles and techniques for computer graphics on modern graphics hardware. Students will gain experience in interactive computer graphics using the OpenGL API. Topics include: 2D viewing, 3D viewing, perspective, lighting, and geometry. This course will introduce students to all aspects of computer graphics including hardware, software and applications. Students will gain experience using a graphics application programming interface (OpenGL) by completing several programming projects.

Contents

1. Introduction to computer graphics & its applications
2. Overview of raster graphics & transformation pipeline
3. Transformations between different coordinate systems which involve modeling coordinate system
4. Device coordinate system,
5. World coordinate system
6. Normalized coordinate system
7. Display window coordinate system & screen coordinate system
8. Graphics output primitives in drawing of lines, polygons, triangles
9. Draw polylines with different line joining methods
10. Attributes of graphics primitives like color, line style & fill style
11. 2D & 3D transformations & viewing
12. Describing & using viewing parameters to change the shape of the object, using viewport to
    change the ratio of clipping window
13. Differences in viewing & modeling transformations
14. Window clipping by Cohen-Sutherl& algorithm

Recommended Texts

1. Donald, H., & Baker, M. P. (2003). *Computer graphics with open*. New Jersey: Prentice Hall.
2. Peter, S., & Steve, M. (2015). *Fundamentals of computer graphics*. Florida: CRC Press.

Suggested Readings

1. Richard, S. W., & Benjamin, L. (2004), *OpenGL superbible*. Philadelphia: Society for Industrial & Applied Mathematics.
2. Samuel, R. B. (2003). *3D computer graphics, a mathematical introduction with open*. Cambridge: Cambridge University Press.

MATH-7154 **Dynamic Inequalities on Time Scales** **3 (3+0)**

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his PhD thesis in 1988 (supervised by Bernd Aulbach) to unify continuous & discrete analysis. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different in nature from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies, & helps avoid proving results twice, once for differential equations & once for difference equations. The general idea is to prove a result for a dynamic equation where the domain of the unknown function is a so-called time scale, which is an arbitrary nonempty closed subset of the reals. By choosing the time scale to be the set of real numbers, the general result yields a result concerning an ordinary differential equation as studied in a first course in differential equations. On the other h&, by choosing the time scale to be the set of integers, the same general result yields a result for difference equations. However, since there are many other time scales than just the set of real numbers or the set of integers, one has a much more general result.

Contents

1. Time scale calculus, Basic definitions, differentiation
2. Examples & applications
3. Integration, chain rule, polynomials
4. Further basic results. Dynamic inequalities
5. Gronwall inequality, Holder & Minkowski's inequalities
6. Jensen's inequality

Recommended Texts

1. Bohner M., & Peterson A. (2001). *Dynamic equations on time scales*. Basel: Birkhauser boston, mass.
2. Bohner M., & Peterson A. (2003). *Advances in dynamic equations on time scales*. Basel: Birkhauser boston, mass.

Suggested Readings

1. Lakshmikatham, V., Bhaskar T. G., & Devi J. V. (2006). *Theory of Set Differential equations in Metric Spaces*. Cambridge*:* Cambridge Scientic.
2. Peter, S., & Steve, M. (2015). *Fundamentals of computer graphics*. Florida: CRC Press.

MATH-7155 **Set-Valued Analysis** **3 (3+0)**

The main objective is to introduce topological spaces and set-valued maps for those who are aspiring to work for their Ph. D. in mathematics. It is assumed that measure theory and metric spaces are already known to the reader. Hence, only a review has been made of metric spaces. At the same time the topics on topological spaces are taken up as long as they are necessary for the discussions on set-valued maps. This course is devoted to a compressed & self-contained exposition of an important part of contemporary mathematics: set-valued analysis. This course covers the fundamentals of mathematical analysis: convergence of sequences & series, continuity, semicontinuity, derivative, integral, sequences & series of functions, uniformity, & the interchange of limit operations for a set-valued mapping. The course aims at familiarizing the students with measurable set-valued functions, continuous, Lipschitz & some special types of selections, fixed point & coincidence theorems, covering set-valued maps, topological degree theory & differential inclusions, Aumann integral & Hukuhara derivative. After successful completion of the course, students are able to apply the theory of set-valued mappings, the notions of tangent and normal cones, fixed point theorems, integral of set-valued functions, the theory of differential inclusions.

Contents

1. Preliminaries
2. Hausdorff-Pompeiu metric
3. Upper & lower semicontinuous
4. Multifunctions
5. Hausdorff-Pompeiu continuity
6. Closed multifunctions, continuous
7. Selections
8. Measurable multifunctions

Recommended Texts

1. Tarafdar, E. U., & Chowdhury, M. S. R. (2008). *Topological methods for set-valued nonlinear analysis* (1st ed.). Singapore: World Scientific Publishing.
2. Lakshmikatham, V., Bhaskar, T. G., & Devi J. V. (2006). *Theory of set differential equations in metric* Spaces (1st ed.). Cambridge: Cambridge Scientic.

Suggested Readings

1. Chen, G., Huang, X., & Yang, X. (2005). *Vector optimization: Set-valued & variational analysis* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Aubin, J. P., & Frankowska, H. (1990). *Set-Valued Analysis* (1st ed.). Basel: Birkhauser Boston.

MATH-7156 Fractional Calculus 3 (3+0)

The subject of fractional calculus came to life over the last few decades. A feature is that engineers & scientists have developed new models that involve fractional differential equations. These models have been applied successfully, e.g., in mechanics (theory of viscoelasticity), bio-chemistry (modelling of polymers & proteins), electrical engineering (transmission of ultrasound waves), medicine (modelling of human tissue under mechanical loads), etc. The mathematical theory seems to be lagging the needs of those applications, but the wealth of applications indeed indicates the truth of the above quote from Heaviside. There are some aspects that can be summarized as the "pure mathematical" side of the problems without taking into consideration those questions that arise in the applications mentioned above, & some that the engineer's point of view without a rigorous mathematical justification of the ideas. This subject attempts to fill the gap between these two approaches. There established mathematically a sound theory of the differential equations that have been shown to be relevant in practice & provide a thorough mathematical analysis. The fundamentals of fractional calculus with its applications is also a goal of this course to provide a solid foundation that may later be used for the construction of efficient & reliable numerical methods for fractional differential equations. A successful development & a thorough underst&ing of such numerical schemes is not possible without such a stable analytical background. The students are assumed to be familiar with classical calculus (differential & integral calculus & the elementary theory of differential equations) to cope this advanced course in the context of theory of Lebesgue integrals.

Contents

1. Introduction; motivation
2. Basics
3. Application of fractional calculus
4. Riemann–Liouville
5. Differential & integral operators
6. Riemann–Liouville integrals, Riemann–Liouville
7. Derivatives, relations between Riemann–Liouville integrals & derivatives
8. Grunwald–Letnikov operators
9. Caputo's approach
10. Definition & basic properties
11. Non-classical representations of Caputo operators

Recommended Texts

1. Diethelm, K. (2010). *The analysis of fractional differential equ*ations. New York: Springer-Verlag Berlin Heidelberg.
2. Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). *Theory & applications of fractional differential equations.* North-Holland: Elsevier.

Suggested Readings

1. Oldham, K. B., & Spanier, J. (2006). *The fractional calculus: Theory & applications of differentiation & integration to arbitrary order*. New York: Dover Books on Mathematics.
2. Aubin, J. P., & Frankowska, H. (1990). *Set-Valued Analysis* (1st ed.). Basel: Birkhauser Boston.

MATH-7157 **Perturbation Methods-I** **3 (3+0)**

The purpose & objective of this course is to learn & exercise the applications of perturbation expansion techniques to the solution of differential equations & approximation of integrals. Approximation expansions are generated in the form of asymptotic series. These may not & often do not converge but, in a truncated form of only two or three terms, provide a useful approximation to the original problem. The techniques, being analytical rather than numerical, provide an alternative to a direct computer solution. Awareness of the perturbation approach is sometimes essential even when a direct numerical approach is adopted. Students, teachers & researchers in applied & engineering mathematics should learn this technique to gaining an underst&ing of most of the powerful perturbation techniques along with their applications. The basic ideas, however, are also applicable to integral equations, integro differential equations, & even to difference equations. In essence, a perturbation procedure consists of constructing the solution for a problem involving a small parameter B, either in the differential equation or the boundary conditions or both, when the solution for the limiting case B = 0 is known. The main mathematical tool used is asymptotic expansion with respect to a suitable asymptotic sequence of functions of B.

Contents

1. Parameter of perturbations
2. Coordinate perturbations, order symbols & gauge functions
3. Asymptotic series & expansions
4. Asymptotic expansion of integrals, integration by parts
5. Laplace’s method & Watson’s lemma, method of stationary phase & method of steepest descent
6. Straightforward expansions & sources of non-uniformity
7. The Duffing equation, small Reynolds number
8. The method of strained coordinates
9. The Lindstedt – Poincare’ method
10. Renormalization method
11. Variation of parameters & method of averaging, examples
12. Method of Multiple scales with examples
13. Flow past a sphere
14. Small parameter multiplying the highest derivative

Recommended Texts

1. Bush, A. W. (1992). *Perturbation methods for engineers & scientists*. Florida: CRC Press.
2. Ali, H. N. (1980). *Introduction to perturbation techniques*. New York: A Willey Inter science Publication John Willey & Sons.

Suggested Readings

1. Giacoglia, G. E. O. (1972). *Perturbation methods in non-linear system*. New York: Springer.
2. Holmes, M. H. (2013). *Introduction to perturbation methods.* New York: Springer.
3. Recent research articles.

MATH-7158 **Perturbation Methods-II** **3 (3+0)**

The purpose & objective of this course is to learn & exercise the applications of perturbation expansion techniques to the solution of differential equations & approximation of integrals. Approximation expansions are generated in the form of asymptotic series. These may not & often do not converge but, in a truncated form of only two or three terms, provide a useful approximation to the original problem. The techniques, being analytical rather than numerical, provide an alternative to a direct computer solution. Awareness of the perturbation approach is sometimes essential even when a direct numerical approach is adopted. Students, teachers & researchers in applied & engineering mathematics should learn this technique to gaining an underst&ing of most of the powerful perturbation techniques along with their applications. The perturbation methods, especially in connection with differential equations, in order to illustrate certain general features common to many examples will be exercised. The basic ideas, however, are also applicable to integral equations, integro differential equations, & even to difference equations. In essence, a perturbation procedure consists of constructing the solution for a problem involving a small parameter B, either in the differential equation or the boundary conditions or both, when the solution for the limiting case B = 0 is known. The main mathematical tool used is asymptotic expansion with respect to a suitable asymptotic sequence of functions of B.

Contents

1. Approximate Solution of Linear Differential Equations
2. Approximate Solution of Nonlinear Differential Equations
3. Perturbation Series
4. Regular & Singular Perturbation Theory
5. Perturbation Methods for Linear Eigen value Problems
6. Asymptotic Matching, Boundary Layer Theory
7. Mathematical Structure of Boundary Layers: Inner, Outer, & Intermediate Limits Higher-Order Boundary Layer Theory
8. Distinguished Limits & Boundary Layers of Thickness
9. WKB Theory Exponential Approximation for Dissipative & Dispersive Phenomena,
10. Conditions for Validity of the WKB approximation, matched Asymptotic
11. Approximations: WKB Solution of inhomogeneous Linear equations
12. Matched Asymptotic Approximation: Solution of the One-Turning-Point Problem

Recommended Texts

1. Bush, A. W. (1992). *Perturbation methods for engineers & scientists*. Florida: CRC Press.
2. Ali, H. N. (1980). *Introduction to perturbation techniques*. New York: John Willey & Sons.

Suggested Readings

1. Giacoglia, G. E. O. (1972). *Perturbation methods in non-linear system*. New York: Springer.
2. Holmes, M. H. (2013). *Introduction to perturbation methods.* New York: Springer.

MATH-7159 **Viscous Fluids-I** **3(3+0)**

This is a first graduate level course intended to cover the fundamentals of fluid mechanics from an advanced point of view, with emphasis on the mathematical treatment of viscosity effects in laminar flows of a Newtonian fluid. We begin with the Navier Stokes equations & some of its exact solutions available in simplified configurations. Attention is given to the Stokes-flow regime of very low Reynolds numbers, flows with wall & free-shear boundaries, & the effects of pressure gradients, heat transfer & compressibility. We also provide an introduction to the phenomena of instability & transition to turbulence. Observations on Taylor-vortex flows between cylinders of comparatively small but variable length are reported, revealing properties unexplained by older theories. The observed flows are classified as follows: (i) the primary mode which is uniquely possible at small values of the Reynolds number *R*, & which usually develops smoothly with increasing *R*; (ii) secondary modes which are possible only above a respective critical value of *R*, & which are shown to manifest predicted behavior as this value is approached.. Two novel & surprising examples of (ii) are reported. A predicted hysteresis phenomenon is confirmed, relating to morphogenesis of the primary mode between two-cell & four-cell forms as the length of the annulus is varied.

Contents

1. Some examples of viscous flow phenomena
2. Properties of fluids
3. Boundary conditions
4. Equation of continuity
5. The Navier stokes equations
6. The energy equation
7. Orthogonal coordinate systems
8. Dimensionless parameters
9. Velocity considerations; two dimensional considerations, & the stream functions
10. Coutte flows, Paisvilleflow, unsteady duct flows
11. Similarity solutions, some exact analytic solution from the papers
12. Introduction; laminar boundary layers equations; similarity solutions
13. Two dimensional solutions
14. Thermal boundary layer
15. Some exposure will also be given from the recent literature appearing in the journals

Recommended Texts

1. White, F. M. (2005). *Viscous fluid flow* (3rd ed.). New York: McGraw Hill Inc.
2. Schlichting, H., & Gertsen, K. (1991). *Boundary layer theory.* New York: Springer.

Suggested Readings

1. Davidson, P. A. (2001). *An Introduction to magnetohydrodynamics*. Cambridge: Cambridge University Press.
2. Holmes, M. H. (2013). *Introduction to perturbation methods.* New York: Springer.

MATH-7160 **Viscous Fluids-II** **3(3+0)**

This is a first graduate level course intended to cover the fundamentals of fluid mechanics from an advanced point of view, with emphasis on the mathematical treatment of viscosity effects in laminar flows of a Newtonian fluid. We begin with the Navier Stokes equations & some of its exact solutions available in simplified configurations. Attention is given to the Stokes-flow regime of very low Reynolds numbers, flows with wall & free-shear boundaries, & the effects of pressure gradients, heat transfer & compressibility. The observed flows are classified as follows: (i) the primary mode which is uniquely possible at small values of the Reynolds number R, & which usually develops smoothly with increasing R; (ii) secondary modes which are possible only above a respective critical value of R, & which are shown to manifest predicted behavior as this value is approached.. Two novel & surprising examples of (ii) are reported. A predicted hysteresis phenomenon is confirmed, relating to morphogenesis of the primary mode between two-cell & four-cell forms as the length of the annulus is varied. We also introduce the phenomena of instability & transition to turbulence.

Contents

1. Introduction
2. The concept of small disturbance stability
3. Linearized stability; parametric effects in the linear stability theory; transition to turbulences
4. Boundary layer equation in plane flow
5. General solution & exact solutions of the boundary layer equations
6. Thermal boundary layers without coupling of velocity field to the temperature field
7. Boundary layer equations for the temperature field
8. Forced convection; effect of Pr number
9. Similar solution of the thermal boundary layers
10. Thermal boundary layer with coupling of velocity field to the temperature field
11. Boundary layer with moderate wall heat transfer
12. Natural convection effect of dissipation

Recommended Texts

1. White, F. M. (2005). *Viscous fluid flow* (3rd ed.). New York: McGraw Hill Inc.
2. Schlichting, H., & Gertsen, K. (1991). *Boundary layer theory.* New York: Springer.

Suggested Readings

1. Davidson, P. A. (2001). *An Introduction to magnetohydrodynamics*. Cambridge: Cambridge University Press.
2. Holmes, M. H. (2013). *Introduction to perturbation methods.* New York: Springer.

MATH-7161 **Fuzzy Algebra** **3(3+0)**

The primary purpose of this course is to introduce students to the important areas of fuzzy set theory & fuzzy logic. No previous knowledge is needed regarding fuzzy set theory or fuzzy logic. But familiarity with classical set theory, & two-valued logic will be helpful. In most real-life applications of any decision making one needs to face many types on uncertainty. While as humans we can deal with this uncertainty with our reasoning prowess it is not clear how to deal with this uncertainty in a system. Fuzzy sets & fuzzy logic gives us one way of representing this uncertainty & reasoning with them. This course is aimed at providing a strong background for the subject. The decomposition theorems of fuzzy sets & the extension principle will be introduced, as well as the use of nonlinear integrals as aggregation tools to deal with fuzzy data. As an indispensable tool in fuzzy decision making, ranking & ordering fuzzy quantities will be discussed.

Contents

1. Introduction, the Concept of Fuzziness, Examples
2. Mathematical Modeling
3. Operations of fuzzy sets
4. Fuzziness as uncertainty
5. Algebra of Fuzzy Sets
6. Boolean Algebra & lattices
7. Equivalence relations & partions
8. Composing mappings
9. Alpha-cuts, Images of alpha-level sets
10. Operations on fuzzy sets
11. Fuzzy Relations, definition & examples
12. Binary Fuzzy relations Operations on Fuzzy relations, fuzzy partitions
13. Fuzzy Semigroups
14. Fuzzy ideals of semigroups, Fuzzy quasi-ideals, Fuzzy bi-ideals of Semigroups

Recommended Texts

1. Hung, T. N. (2005). *A first course in fuzzy logic* (3rd ed.). Boca Raton: Chapman & Hall/CRC.
2. Ganesh, M. (2006). *Introduction to fuzzy sets & fuzzy lo*gic (1st ed.). New Jersey: Prentice-Hall.

Suggested Readings

1. Mordeson, J. N., & Malik, D. S. (1998). *Fuzzy commutative algebra* (1st ed.). New York: Singapore: World Scientific.
2. Mordeson, J. N., Malik, D. S., & Kuroki, N. (2003). *Fuzzy semigroups* (1st ed.). New York: Springer-Verlage Heidelberg.

MATH-7162 **Minimal Surfaces** **3(3+0)**

The main objective is to present to the students & to the young researchers how tools from differential geometry & analysis of the partial differential equations can be combined to obtain interesting, new results in both fields. The Minimal surfaces theory & geometric analysis are very active topics in Brazil. These theories are quite advanced & expect to spur developments in new areas. For example, the Allen-Cahn equation which models two phases transitions is a counterpart of the minimal surface equation in semilinear elliptic partial differential equations. Since the resolution of the De Gorgi conjecture by the group of nonlinear PDEs in Chile there has been growing interest in geometric aspects of semilinear elliptic equations. Regularity theory of a minimizer of an elliptic functional is at the origin of the subject, beginning with the work of Modica. The De Giorgi's conjecture is related to the Bernstein problem in minimal surface theory.

Contents

1. Regular Surfaces
2. Differentiable functions on surfaces
3. The tangent plane
4. Geometric definition of area
5. Gaussian & mean curvature
6. Curvature in local coordinates
7. Ruled & minimal surfaces
8. Historical survey & introduction to the theory of minimal surfaces
9. Basic minimal surfaces properties
10. Topological & physical properties
11. Stable & unstable minimal surfaces
12. Two dimensional minimal surfaces in three dimensional space
13. Helecoid
14. Catenoid & conoid
15. Harmonic approximation to area
16. Nambu-Goto action
17. Compactness
18. Singularities
19. Topological applications

Recommended Texts

1. Dierkes, U. (2005). *Minimal surfaces* (2nd ed.). New York: Springer.
2. Robert, O. (2002). *A survey of minimal surfaces*. New York: Dover Pub.

Suggested Readings

1. Tobias, H. C., & William, P. M. (2011). *A course in minimal surfaces.* Michigan: American Mathematical Society.
2. Ulrich, D., Stefan, H., & Anthony, J. T. (2010). *Regularity of minimal surfaces*. New York: Springer-Verlag Berlin.

MATH-7163 **Riemannian Geometry** **3 (3+0)**

Riemannian geometry is the branch of differential geometry that studies riemanian manifolds, smooth manifold with a riemanian metric that is with an inner product on the tangent space at each point that varies smoothly from point to point. Since Euclidian geometry is the study of flat space, between each point of point there is a unique line segment which is the shortest curve. This line segment can be ext&ed to lines of infinite length. This course with main purpose is to introduce the beautiful theory of Riemannian geometry, a still very active area of mathematical research. The study of Riemannian geometry is rather meaningless without some basic knowledge on Gaussian geometry i.e. the geometry of curves & surfaces in 3-dimensional Euclidean space. The course will begin with an overview of Riemannian manifolds including such basics as geodesics, curvature, and the exponential map. As examples, the course will emphasize things like spaces of constant Grassmanians, Lie groups, and symmetric spaces. Then the course will cover topological and geometric consequences of curvature such as the Cartan-Hadamard theorem. The course will conclude with a brief sketch of some more advanced topic, such as comparison geometry, hyperbolic structures on surfaces (Teichmüller space).

Contents

1. Definitions & examples of manifolds
2. Tangents
3. Coordinate vector fields
4. Tangent spaces, Dual spaces
5. Algebra of tensors, Vector fields
6. Tensor fields, Integrals curves
7. Affine connections & Christoffel symbols
8. Covariant differentiation of tensor fields
9. Geodiscs equations, Curve on manifold, Parallel transport
10. Lie transport, Lie derivatives & Lie brackets
11. Geodisc deviationDifferential form
12. Introduction to integration theory on manifolds
13. Riemannian curvature tensor
14. Ricci tensor & Ricci scalar
15. Killing equations & killing vector fields

Recommended Texts

1. Bishop, R. L., & Goldberg, S. I. (1980). *Tensor analysis on manifolds* (1st ed.). New York: Dover Publication.
2. Carmo, M. P. (1992). *Riemannian geometry* (1st ed.). Boston Switzerland: Birkahauser.

Suggested Readings

1. Langwitz, D. (1970). *Differential & riemannain geometry*. Cambridge: Academic Press.
2. Abraham, R., Marsden, J. E., & Ratiu, T. (1993). *Manifolds tensor analysis & applications*. New York: Addison-Wesley.

PhD
COURSES

MATH-8101 **Lie Algebras** **3 (3+0)**

In this course, Lie algebras are related to Lie groups, & both concepts have important applications to geometry & physics. Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, & describe parts of the classification mentioned above, especially the parts concerning root systems & Dynkin diagrams. The Lie algebras considered in this course will be finite dimensional vector spaces over endowed with a multiplication which is almost never associative are different in general. Students will learn how to utilize various techniques for working with Lie algebras, & they will gain an underst&ing of parts of a major classification result. Lie groups & Lie algebras have become essential to many parts of mathematics & theoretical physics, with Lie algebras a central object of interest in their own right.

Contents

1. Definitions of Lie algebras
2. Examples of Lie algebras
3. Ideals & quotients simple
4. Solvable & nilpotent Lie algebras
5. Radical of a Lie algebra
6. Semisimple Lie algebras
7. Engel's nilpotency criterion
8. Lie's & Cartan theorems
9. Jordan-Chevalley decomposition
10. Killing forms
11. Criterion for semi simplicity
12. Product of Lie algebras
13. Classification of Lie algebras upto dimension
14. Applications of Lie algebras

Recommended Texts

1. Karin, E., & Mark, J.W. (2006). *Introduction to lie algebras*. New York: Springer.
2. Humphreys, J. E. (2003). *Lie groups, Lie algebras, & representations: An elementary introduction*. New York: Springer.

Suggested Readings

1. Neill, B.O. (1983). *Semi-riemannian geometry*. Cambridge: Academic Press.
2. Stephani, H., Kramer, D., MacCallum, M., Hoenselaers, C., & Herlt, E. (1980). *Exact solutions of einstein's fields equations*. Cambridge: Cambridge University Press.
3. Lepowsky, J., & Mecollum, G.W. (1974). *Elementary lie algebra theory*. New Haven: Yale University.

MATH-8102 Numerical Solutions of Integral Equations 3 (3+0)

The course deals with integral equations, their origin, properties & solutions, both approximate & numerical.Integral equations occur naturally in many fields of science & engineering. A computational approach to solve integral equation is an essential work in scientific research. Integral equation is encountered in a variety of applications in many fields including continuum mechanics, potential theory, geophysics, electricity & magnetism, kinetic theory of gases, hereditary phenomena in physics & biology, renewal theory, quantum mechanics, radiation, optimization, optimal control systems, communication theory, mathematical economics, population genetics, queuing theory, medicine, mathematical problems of radiative equilibrium, the particle transport problems of astrophysics & reactor theory, acoustics, fluid mechanics, steady state heat conduction, fracture mechanics, & radiative heat transfer problems. Integral equations can be viewed as equations which are results of transformation of points in a given vector spaces of integrable functions by the use of certain specific integral operators to points in the same space. If, in particular, one is concerned with function spaces spanned by polynomials for which the kernel of the corresponding transforming integral operator is separable being comprised of polynomial functions only, then several approximate methods of solution of integral equations can be developed. This course is useful for for underst&ing the numerical solutions of physical systems developed in integral forms.

Contents

1. Numerical & approximate solutions of Fredholm integral equitation of the second kind (both linear & nonlinear)
2. Approximation of integral operators & quadrature methods
3. Nystrom method, Method of degenerate kernels.
4. Collectively compact operator approximations
5. Numerical methods of volterra integral equations
6. Methods of collocation
7. Galerkin, moments, & Spline approximations for integral equations
8. Iterative methods for linear & nonlinear integral equations
9. Eigenvalue problems

Recommended Texts

1. Kendall, E.A. (2001). *The numerical solution of integral equations of the second kind*. Cambridge: Cambridge University Press.
2. Michael, A.G. (2001). *Numerical solution of integral equations*. New York: Springer.

Suggested Readings

1. Constanda, C., Doty, D., & Hamill, W. (2016). *Boundary integral equation methods & numerical solutions*. (1st ed.). New York: Springer.
2. Abdul, M. W. (2011). *Linear & nonlinear integral equations methods & application*. New York: Springer.

MATH-8103 Multiresolution Analysis in Geometric Modeling 3 (3+0)

Freeform curves, surface and solids are generally represented in B-spline basis. Various geometric quantities, such as control points, knots and weights must be specified. Controlling the shape of an object under complex deformations by manipulating the control points directly is often difficult. The movement of control points gives an indication of the resulting deformation, but being extraneous to the object, the control points do not allow for precise control of the shape. In addition, large deformations of complex objects with many details to be preserved become nearly impossible without any" higher level" control mechanisms. User-friendly shape-control tools, therefore, generally make use of modeling techniques that integrate constraints. This course surveys the state-of-the-art of geometric modelling techniques that integrate constraints, including direct shape manipulation, physics-based modelling, solid modelling & freeform deformations as well as implicit surface modelling. It focuses on recent advances of multiresolutionmodelling of shapes under constraints. This course also deals with the subdivision algorithms for free-form curves, surfaces & volumes. It plays a significant role in integrating computers & industry.

Contents

1. Interpolatory subdivision schemes, Approximating Schemes
2. Types of Subdivision scheme for curves & surfaces
3. The univariate stationary case, non-stationary univariateinterpolatory schemes exact for exponentials
4. Tensor product interpolatory schemes for surfaces
5. The butterfly scheme
6. Analysis of convergence & smoothness by the formalism of Laurent polynomials: introduction analysis of univariate schemes, analysis of bivariate scheme with factorizable symbols
7. A non-stationary subdivision scheme for curve interpolation
8. A non-stationary subdivision scheme for generalizing trigonometric spline surfaces to arbitrary meshes
9. A non-stationary uniform tension controlled interpolating 4-point scheme reproducing conics

Recommended Texts

1. Armin, I., Ewald, Q., & Michael, F. S. (2002). *Tutorials on multiresolution in geometric modeling. Summer school lecture notes.* New York: Springer-Berlin.
2. Gerald, F., Josef, H., & Myung, S. (2002). *Handbook of computer aided geometric design*. The Netherlands: Elsevier Science.

Suggested Readings

1. Sweldens, W., & Schroder, P. (2005). *Building your own wavelet at home*, online lecture notes.
2. Abdul, M. W. (2011). *Linear & nonlinear integral equations methods & application*. New York: Springer.

MATH-8104 **Advanced Graph Theory** **3 (3+0)**

Advanced Graph Theory focuses on problem solving using the most important notions of graph theory with an in-depth study of concepts on the applications in the field of computer science. This course provides an in-depth understanding of Graphs and fundamental principles and models underlying the theory, algorithms, and proof techniques in the field of Graph Theory. Emerging applications of Graph Theory in Computer Science domain will be covered for significant impact. Upon completing this course, students will have intimate knowledge about how the graph theory play an important role to solve the technology driven and research-oriented problems. Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among other fields, to solve problems that can be modelled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology, & combinatorics. Even though some of the problems in graph theory can be described in an elementary way, many of these problems represent a challenge to many researchers in mathematics.

Contents

1. Graph Spectrum Theory
2. Matrices associated to a graph, the spectrum of various graphs, diameter
3. Regulargraphs, Spanning trees, Complete graphs
4. Complete bipertite graphs, Calay graphs
5. Some Modules on the Applications of Spectral Graph Theory
6. Introduction to spectral geometry of graphs
7. Courant-Fischer theorem & graph colorings
8. Inequalities & bounds on eigenvalues; graph approximations
9. Cheeger's inequalities; Diffusion on graphs; Discretizations of heat kernels
10. Energy of Graph, Laplacian Energy, Signless Energy, Distance Energy
11. Normalized Laplacian Energy
12. He-matrix for Honey Comb Graph
13. Honeycomb graph, He matrix

Recommended Texts

1. Godsil, C., & Royle, G., (2004). *Algebraic graph theory*. New York: Springer.
2. Edgar, G., & Michael M. P. (2005). *Discrete mathematics with graph theory* (3rd ed.).New Jersey: Prentice Hall.

Suggested Readings

1. Biggs, N. L. (1993). *Algebraic graph theory*. Cambridge: Cambridge University.
2. Sweldens, W., & Schroder, P. (2005). *Building your own wavelet at home*, online lecture notes.

MATH-8105 **Strict Convexity** **3 (3+0)**

The main aim of this course is to underst& many of the sub-stational properties of optimization. Convex functions play an important role in many fields of mathematics such as optimization, control theory, operations research, geometry, differential equations, functional analysis etc., as well as inapplied sciences & practice, e.g. in economics, finance.They have a lot of interesting & fruitful properties, e.g. continuity& differentiability properties or the fact that a local minimum turns outto be a global minimum etc. They even allow to establish a proper & general theory of convex functions, moreover together with convex sets, the so-called theory of Convex Analysis, which is important not only for itselfbut also for its many applications in the theory of convex & also non-convexoptimization. Convexity of sets & functions are extremely simple notions to define, so it maybe somewhat surprising the depth & breadth of ideas that these notions give riseto. It turns out that convexity is central to a vast number of applied areas, including Statistical Mechanics, Thermodynamics, Mathematical Economics, & Statistics, & that many classical inequalities, including Holder, Minkowski, Jensen, Hermite-Hadamard, majorization inequalities are related to convexity.

Contents

1. Locally covex spaces
2. Banach spaces
3. Basic theorems of linear functional analysis
4. Strict convex spaces
5. Product spaces
6. Quotient spaces
7. Strict convexity
8. Interpolation & strict convexity
9. Modulus of convexity
10. Approximation theory
11. Strict convexity & approximation theory
12. Fixed point theory
13. Strict convexity & fixed point theory

Recommended Texts

1. Lars, H. (2006). *Notions of convexity*. Switzerland: Birkhäuser Boston.
2. Constantin, N., & Lars-Erik, P. (2005). *Convex functions & their applications*. New York: Springer.

Suggested Readings

1. Istratescue, V. I. (1984). *Strict convexity & complex strict, convexity*. Florida: CRC Press.
2. Diestel, J. (1975). *Geometry of Banach spaces*. New York: Springer.

MATH-8106 **Applications of Inequalities** **3 (3+0)**

The main purpose of this course is the study of some mathematical inequalities. Furthermore, the applications & integral operators of the inequalities involving special functions are to be discussed. Mathematics is not always about "equals". We compare one thing with the other thing in mathematics & in other branches of science. In mathematics, an inequality is a relation that holds between two values or quantities when they are different. The theory of inequalities has been recognized as one of the central areas of mathematical analysis. During the past several years, the usefulness & applications of mathematical inequalities in various branches of mathematics as well as in other areas of science are well established. It is a fast-growing discipline with ever increasing applications in scientific fields such as mathematical economics, game theory, mathematical programming, control theory, variational methods, operational research, probability & statistics. This growth resulted in the appearance of the theory of inequalities as an independent domain of mathematical analysis. Inequalities for the special functions appear infrequently in the literature. Some of these inequalities are closely related to those presented here. Inequalities for the ratio of confluent hypergeometric functions are available in the literature. The formulas are very important, as they include expansions for many transcendent expressions of mathematical physics in series of the classical orthogonal polynomials, Laguerre, Hermite functions

Contents

1. Grüss type inequalities, Chebychev's type inequalities
2. Ostrowski's inequalities
3. Hardy-type Inequalities
4. Jensen's & related inequalities,
5. Hadamard's inequalities,
6. Applications of inequalities involving gamma & beta functions
7. Applications of inequalities involving beta functions
8. Introduction to Lp-spacesinvolving special functions
9. Boundedness of integral operators involving some special functions
10. Miscellaneous inequalities

Recommended Texts

1. Pachpatte, B. G. (2005). *Mathematical inequalities*. The Netherlands: Elsevier.
2. Kilbas, A. A., Srivastava, H. M., & Trujillo, J. J. (2006). *Theory & applications of fractional differential equations*. The Netherlands: Elsevier.

Suggested Readings

1. Mitrinovic, D. S., Pečarić, J., & Fink, A. M. (1993). *Classical & new inequalities in analysis.* The Netherl&s: Kluwer Academic Publishers.
2. Pečarić, J., Proschan, F., & Tong, Y. C. (1992). *Convex functions, partial orderings & statistical applications*. USA: Academic Press.
3. Okikiolu, G. O. (1971). *Aspects of the theory of bounded linear operators*. Cambridge: Academic Press.

MATH-8107 Propagation of Waves in Different Media 3 (3+0)

This course for underst&ing of Electromagnetic waves is crucial for developing a wide range of practical electrical engineering systems including telecommunication networks, high speed electronics, & photonics systems. This course will cover core concepts of electromagnetic wave theory with emphasis on transmission lines & propagation of plane electromagnetic waves in various media. Each covered concept will be augmented with discussions on industrial & research applications. The propagation & reflection of plane waves for Pressure waves (P-wave) or Shear waves (SH or SV-waves) are phenomena that were first characterized within the field of classical seismology, & are now considered fundamental concepts in modern seismic tomography. The analytical solution to this problem exists & is well known. The frequency domain solution can be obtained by first finding the Helmholtz decomposition of the displacement field, which is then substituted into the wave equation. From here, the plane wave Eigen-modes can be calculated. The analytical solution of SV-wave in a half-space indicates that the plane SV wave reflects back to the domain as a P & SV waves, leaving out special cases. The angle of reflected SV wave is identical to the incidence wave, while the angle of reflected P wave is greater than the SV wave.

Contents

1. Waves in infinite Media: Wave types
2. The governing equations, dilatational & distortional waves
3. Plane waves, waves generated by body forces
4. Certain classical solutions
5. Simple SH wave source, cavity source problems
6. Harmonic dilatational waves from a spherical cavity
7. Dilatational waves from a step pulse
8. General case of dilatational waves with spherical symmetry
9. Harmonic waves from a cylindrical cavity
10. Transient waves from a cylindrical cavity
11. Propagation in a semi-infinite media
12. Propagation & reflection of plane waves in a half-space
13. Governing equations
14. Waves at oblique incidence& waves at grazing incidence
15. Surface waves, wave reflection under mixed boundary conditions

Recommended Texts

1. Magrab, E. B. (2010). *Vibrations of elastic structural members: Mechanics of structural systems*. New York: Springer.
2. Kolskey, H. (2003). *Stress waves in solids, Dover phoenix editions*. New York: Dover Publications.

Suggested Readings

1. Marc, A. M. (1994). *Dynamic behavior of materials*. New York: Wiley-Interscience.
2. Karl, F. G. (1991). *Wave motion in elastic solids*. New York: Dover Publications.
3. Achenbach, J. D. (1984). *Wave propagation in elastic solids*. The Netherlands: Elsevier.

MATH-8108 Scattering and Diffraction of Elastic Waves 3(3+0)

Elastic waves are generated whenever a transient stress imbalance is produced within or on the surface of an elastic medium. Almost any sudden deformation or movement of a portion of the medium results in such a source. Elastic wave is an elastic disturbance that propagates in a solid, liquid, or gaseous medium. Examples of elastic waves include the waves generated in the earth's crust during earthquakes & sound waves & ultrasonic waves in liquids & gases. When elastic waves propagate, the energy of elastic deformation is transferred in the absence of a flow of matter, which occurs only in special cases, such as during an acoustic wind. Every harmonic elastic wave is characterized by the amplitude & vibration frequency of the particles of the medium, a wavelength, phase & group velocities, & a law governing the distribution of displacements & stresses over the wave front. The objective of this course is, theoretical learning for detection of cracks with the aid of ultrasonics & is an important non-destructive evaluation (NDE) technique. The corresponding theoretical problem of the scattering of elastic waves by cracks has also been studied by scientists working in many different fields. Contributions have come from such diverse areas as geophysics, applied mathematics, electrical engineering, & continuum mechanics. Many of the results obtained by workers in those fields are also of interest to the NDE community & for that reason a review is presented here of current results & profitable directions for future research.

Contents

1. SH waves source-method of steepest descent
2. The SH wave- formal solution
3. SH wave source solution by steepest decent
4. Surface source problems, waves from harmonic, normal line force
5. Transient normal loading on a half-space, waves in layered media
6. Two semi-infinite media in contact-plane waves
7. Love waves, experimental studies on waves in semi-infinite media, surface waves on a half-space, other Studies on surface waves
8. Scattering of waves by cavities
9. Scattering of SH waves by a cylindrical cavity
10. Scattering of Compressional waves by a spherical obstacle
11. Diffraction of plane waves
12. Discussion of greens function approach, the Somerfield diffraction problem

Recommended Texts

1. John, G. H. (2001). *Linear elastic waves: Cambridge texts in applied mathematics*. Cambridge: Cambridge University Press.
2. Magrab, E. B. (2010). *Vibrations of elastic structural members (Mechanics of structural systems)*. New York: Springer.

Suggested Readings

1. Kolskey, H. (2003). *Stress waves in solids*. New York: Dover Publications.
2. Karl, F. G. (1991). *Wave motion in elastic solids*. New York: Dover Publications.
3. Marc, A. M. (1994). *Dynamic behavior of materials*. New York: Wiley-Interscience.

MATH-8109 **Lattice Theory** **3 (3+0)**

A lattice is an abstract structure studied in the mathematical subdisciplines of order theory & abstract algebra. It consists of a partially ordered set in which every two elements have a unique supremum (also called a least upper bound or join) & a unique infimum (also called a greatest lower bound or meet). Lattices can also be characterized as algebraic structuressatisfying certain axiomatic identities. Since the two definitions are equivalent, lattice theory draws on both order theory & universal algebra. Semilattices include lattices, which in turn include Heyting & Boolean Algebra. These "lattice-like" structures all admit order-theoretic, as well as algebraic descriptions. Lattice theory is based on a single undefined relation, the inclusion relation & considered as younger brother of Group Theory. It is often said that mathematics is a language. If so, group theory provides the proper vocabulary for discussing symmetry. In the same way, lattice theory provides the proper vocabulary for discussing order, & especially systems which are in any sense hierarchies. One might also say that just as group theory deals with permutations, so lattice theory deals with combinations. One difference between the two is that whereas our knowledge of group theory has increased by not more than fifty per cent in the last thirty years, our knowledge of lattice theory has increased by perhaps two hundred per cent in the last ten years. Lattice theory is the study of sets of objects known as lattices. It is an outgrowth of the study of Boolean algebra & provides a framework for unifying the study of classes or ordered sets in mathematics. The course aims to provide the students with a thorough knowledge of the lattice theory, & its applications to mathematics.

Contents

1. Elementary Concepts
2. Definition of lattice
3. Some algebraic concepts
4. Polynomials
5. Identities
6. Inequalities
7. Free lattices
8. Special elements
9. Distributive lattices
10. Characterization & representation theorems
11. Polynomials & freeness
12. Congruence relations.
13. Boolean algebra

Recommended Texts

1. Blyth, T. S., & Birkhoff, G. (2005). *Lattices & ordered algebraic structures*. New York: Springer.
2. Davey, B. A., & Priestley, H. A. (2002). *Introduction to lattices & order*. Cambridge: Cambridge University Press.

Suggested Readings

1. Grazer, G. (1971). *Lattice theory*. New York: W.H. Freeman & Company.
2. Donnellan, T. (1968). *Lattice theory*. The Netherlands: Elsevier Science Ltd.

MATH-8110 Gravitational Collapse & Black Holes 3 (3+0)

The gravitational field in the vicinity of a collapsing object is so enormous that matter cannot escape. In fact, because light photons have equivalent mass ($m = E/c^2$), even light is prevented from leaving the object. For this reason, these curious objects are called black holes. Gravitational collapse is a fundamental mechanism for structure formation in the universe. Over time an initial, relatively smooth distribution of matter will collapse to form pockets of higher density, typically creating a hierarchy of condensed structures such as clusters of galaxies, stellar groups, stars & planets. A star is born through the gradual gravitational collapse of a cloud of interstellar matter. This course provides an introduction to the dynamical aspects of the space-time geometry in the form of gravitational waves, & of the properties of the gravitationally collapsed objects (black holes). A basic underst&ing of black holes & gravitational waves including the ability to do computations of the most important effects, within the framework of general relativity.

Contents

1. Singularity
2. Kinds of singularity
3. Black holes
4. Schwarzschild singularity
5. The Schwarzschild solutions in other coordinates systems
6. Schwarzschild solution as a black hole
7. Gravitational collapse
8. Collapse to a black hole
9. The evolutionary phases of a spherically symmetric stars
10. The critical mass of star
11. Gravitational collapse of spherically symmetric dust
12. Rotating black holes
13. The Kerr solution, gravitational collapse
14. The possible life history of a rotating star
15. Some properties of black holes

Recommended Texts

1. Stephani, H. (2004). *Relativity, an introduction to special & general relativity* (3rd ed.). UK: Cambridge University Press.
2. Rindler, W. (2007). *Relativity, special & cosmology* (2nd ed.). Oxford: Oxford University Press.

Suggested Readings

1. Hartle, J. B. (2006). *Gravity, an introduction to Einstein's general relativity* (1st ed.). Canada: Pearson Education.
2. Grazer, G. (1971). *Lattice theory*. New York: W.H. Freeman & Company.

MATH-8111 **Spectral Theory in Hilbert Spaces–I** **3 (3+0)**

The aim of this course is to introduce spectral theory of Hilbert space operators. Its emphasis is on recent aspects of theory & detailed proofs, with the primary goal of offering modern introductory lectures for a first graduate course in the subject. Spectral Theory of Operators on Hilbert Spaces is addressed to an interdisciplinary audience of graduate students in mathematics, statistics, economics, engineering, & physics. It will also be useful to working mathematicians using spectral theory of Hilbert space operators, as well as for scientists wishing to apply spectral theory to their field. We begin with a direct proof of the spectral theorem for bounded operators. In order to prepare for a more abstract approach, we give a brief introduction to Banach algebras in the second section, culminating in the Gelf&-Naimark theorem. Using this theory we give an alternative version of the spectral theorem & use this to give the final form that for unbounded operators. Later, partly motivated by the needs of quantum theory (where non bounded self-adjoint operators appear as observables), the theory was generalised to operators on Hilbert space, which are not necessarily bounded. For such operators, the spectrum need no longer be compact & eigenvectors need no longer exist so that the final form of the theorem is necessarily more abstract & complex.

Contents

1. The concept of Hilbert spaces
2. Finite dimensional Euclidean spaces
3. The specific geometry of Hilbert spaces
4. Subspaces, Orthogonal subspaces. Bases
5. Polynomial bases in L^2 Space, Isomorphisms, Linear operators, bilinear forms
6. Adjoint operators, projection operators
7. The Fourier-Planchereloperators, general theory of linear operators
8. Adjoint operators (general case), differentiation operators in L^2 Spaces
9. Multiplication operators in L^2 Spaces, Closed linear operators
10. Invariant subspaces of a linear operator
11. Eigenvalues of a linear operator
12. The Spectrum of a linear operator
13. The spectrum of a self-adjoint operators

Recommended Texts

1. Helmberg, G. (2008). *Introduction to spectral theory in Hilbert spaces*. New York: Dover Publications.
2. Debnath, L., & Mikusinski, P. (2005). *Introduction to Hilbert spaces with applications* (3rd ed.). India: Saurabh Printers Noida.

Suggested Readings

1. Riesz, F., & Nagay, B. S. (1955). *Functional analysis*. New York: Ungar Publishing Co.
2. Akhiezer, N. I., & Glazman, I. M. (1993). *Theory of linear operators in Hilbert spaces.* New York: Dover Publications.
3. Rudin, W. (1991). Functional analysis (2nd ed.). New York: McGraw-Hill.

MATH-8112 Spectral Theory in Hilbert Spaces–II 3 (3+0)

The goal of spectral theory, broadly defined, can be described as trying to “classify” all linear operators, & the restriction to Hilbert space occurs both because it is much easier – in fact, the general picture for Banach spaces is barely understood today, & because many of the most important applications belong to this simpler setting. This may seem like luck, but recall that Hilbert spaces are distinguished among Banach spaces by being most closely linked to plane (& space) Euclidean geometry, & since Euclidean geometry seems to be a correct description of our universe at many scales, it is not so surprising, perhaps, that whenever infinite-dimensional arguments are needed, they should also be close to this geometric intuition. The emphasis of the course is on developing a clear & intuitive picture, & we intend a leisurely pace, with frequent asides to analyze the theory in the context of particularly important examples.

Contents

1. Spectral analysis of compact linear operators
2. Compact linear operators, weakly converging sequences
3. The spectrum of a compact linear operator
4. The spectral decomposition of a compact self-adjoint operator
5. Fredholm integral equations, spectral analysis of bounded linear operators
6. The order relation for bounded self-adjoint operators
7. Polynomials in a bounded linear operator
8. Continuous functions of a bounded self-adjoint operator
9. Step functions of a bounded self-adjoint operator
10. The spectral decomposition of a bounded self-adjoint operator
11. Functions of a unitary operator
12. The spectral decomposition of a unitary operator & the spectral decomposition of a bounded normal operator, Spectral analysis of unbounded self-adjoint operators, the Cayley transform
13. The spectral decomposition of an unbounded self-adjoint operator, limit points of a spectrum
14. Perturbation of the spectrum by the addition of a completely continuous spectrum
15. Continuous perturbation, analytic perturbations

Recommended Texts

1. Helmberg, G. (2008). *Introduction to spectral theory in Hilbert spaces*. New York: Dover Publications.
2. Debnath, L., & Mikusinski, P. (2005). *Introduction to Hilbert spaces with applications* (3rd ed.). India: Saurabh Printers Noida.

Suggested Readings

1. Riesz, F., & Nagay, B. S. (1955). *Functional analysis*. New York: Ungar Publishing Co.
2. Akhiezer, N. I., & Glazman, I. M. (1993). *Theory of linear operators in Hilbert spaces.* New York: Dover Publications.
3. Rudin, W. (1991). Functional analysis (2nd ed.). New York: McGraw-Hill.

MATH-8113 **Multivariate Analysis-I** **3 (3+0)**

Multivariate analysis (MVA) is based on the principles of multivariate statistics, which involves observation & analysis of more than one statistical outcome variable at a time. Typically, MVA is used to address the situations where multiple measurements are made on each experimental unit & the relation among these measurements & their structures are important. MVA once solely stood in the statistical theory realms due to the size, complexity of underlying data set & high computational consumption. With the dramatic growth of computational power, MVA now plays an increasingly important role in data analysis & has wide application in omics fields. The course is designed to underst& the statistical analysis of the data collected on more than one (response) variable. These variables may be correlated with each other, & their statistical dependence is often taken into account when analyzing such data. This consideration of statistical dependence makes multivariate analysis somewhat different in approach & considerably more complex than the corresponding univariate analysis, when there is only one response variable under consideration.

Contents

1. Introduction
2. Some multivariate problems & techniques
3. Thedata matrix. Summary statistics
4. Normal distribution theory
5. Characterization & properties
6. Linear Forms
7. The Wishart distribution. The Hotelling T_2-dustribution
8. Distributions related to the multionormal
9. Estimation & Hypothesis testing
10. Maximum likelihood estimation & other techniques
11. The Behrens-Fisher problem.Simultaneous confidence intervals.Multivariate hypothesis testing
12. Design matrices of degenerate rank. Multiple correlation
13. Least squares estimation. Discarding of variables

Recommended Texts

1. Alvin C. R. (2002). *Methods of multivariate analysis* (2nd ed.). United States: Wiley-Interscience.
2. Joseph, F. H., William, C. B., Barry, J. B., & Rolph, E. A. (2009). *Multivariate data analysis* (7th ed.). New Jersey: Prentice Hall.

Suggested Readings

1. Mardia, K. V., Kent, J. T., & Bibby, J. M. (1982). *Multivariate analysis*. London: Academic Press.
2. Kshirsagar, A. M. (1972). *Multivariate analysis*. New York: Marcell Dekker.

MATH-8114 **Multivariate Analysis-II** **3 (3+0)**

Multivariate Analyses is conceptualized by tradition as the statistical study of experiments in which multiple measurements are made on each experimental unit & for which the relationship among multivariate measurements & their structure are important to the experiment's underst&ing. For instance, in analyzing financial instruments, the relationships among the various characteristics of the instrument are critical. In biopharmaceutical medicine, the patient's multiple responses to a drug need be related to the various measures of toxicity. Multivariate analysis can be complicated by the desire to include physics-based analysis to calculate the effects of variables for a hierarchical "system-of-systems". Often, studies that wish to use multivariate analysis are stalled by the dimensionality of the problem. This is a course in multivariate statistical analysis, for students interested in quantitative methods of marketing research & more generally, for students of sciences. The aim of the course is to explore multivariate techniques used in modern marketing practice & in wider social research. Emphasis will be placed on case studies of marketing practice & on the practical application of the methods discussed.

Contents

1. Principal component analysis
2. Definition & properties of principal components
3. Testing hypotheses about principal components
4. Correspondence analysis, discarding of variables
5. Principal component analysis in regression
6. Factor analysis, the factor model
7. Relationships between factor analysis & principal component analysis
8. Canonical correlation analysis
9. Dummy variables & qualitative data
10. Qualitative & quantitative data, discriminant analysis
11. Discrimination when the populations are known
12. Fisher’s linear discriminant function
13. Discrimination under estimation, Multivariate analysis of variance
14. Formulation of multivariate one-way classification
15. Testing fixed contrasts.Canonical variables & test of dimensionality. Two-way classification

Recommended Texts

1. Alvin C. R. (2002). *Methods of multivariate analysis* (2nd ed.). United States: Wiley-Interscience.
2. Joseph, F. H., William, C. B., Barry, J. B., & Rolph, E. A. (2009). *Multivariate data analysis* (7th ed.). New Jersey: Prentice Hall.

Suggested Readings

1. Mardia, K. V., Kent, J. T., & Bibby, J. M. (1982). *Multivariate analysis*. London: Academic Press.
2. Kshirsagar, A. M. (1972). *Multivariate analysis*. New York: Marcell Dekker.

MATH-8115 **Spacetimes Foliations** **3 (3+0)**

The main aim of this course is to study formal derivation of general relativity as a flat space-time theory with the inclusion of temporal foliation. The term foliation is used to describe a situation where the relevant Lorentz manifold (a (p+1)-dimensional spacetime) has been decomposed into hypersurfaces of dimension p, specified as the level sets of a real-valued smooth function (scalar field) whose gradient is everywhere non-zero; this smooth function is moreover usually assumed to be a time function, meaning that its gradient is everywhere time-like, so that its level-sets are all space-like hypersurfaces. In deference to st&ard mathematical terminology, these hypersurfaces are often called the leaves of the foliation. Note that while this situation does constitute a codimension-1 foliation in the st&ard mathematical sense, examples of this type are actually globally trivial; while the leaves of a (mathematical) codimension-1 foliation are always locally the level sets of a function, they generally cannot be expressed this way globally, as a leaf may pass through a local-trivializing chart infinitely many times, & the holonomy around a leaf may also obstruct the existence of a globally-consistent defining functions for the leaves. The important issue is the Hamiltonian formulation of gravitation, in which case the theorem of Geroch seems to be yet another "no-go" theorem for the necessity of applying deeper topological methods, such as foliations & cobordism, to the problems of spacetime structure.

Contents

1. Foliation
2. Foliation in relativity
3. Foliation & frame of references
4. Hamiltonian formalism & Hamiltonian equation
5. Qadir & Siddiqui flat foliation
6. Flat foliation of Schwarzschild & Rieznor-Nordstormspacetimes by spacelikehypersurfaces
7. Geodesics & foliation of extreme

Recommended Texts

1. Stephani, H. (2004). *Relativity, an introduction to special & general relativity* (3rd ed.). Cambridge: Cambridge University Press.
2. Rindler, W. (2007). *Relativity, special, & cosmology* (2nd ed.). Oxford: Oxford University Press.

Suggested Readings

1. Hartle, J. B. (2006). *Gravity, an introduction to Einstein's general relativity* (1sted.). Canada: Pearson Education.
2. Kshirsagar, A. M. (1972). *Multivariate analysis*. New York: Marcell Dekker.

MATH-8116 **Teleparallel Theory of Gravity**

In this course, we examine theories of teleparallel gravity in detail, & explore their relation to a whole spectrum of alternative gravitational models, discussing their position within the hierarchy of Metric Affine Gravity models. Consideration of alternative gravity models is motivated by a discussion of some of the problems of modern-day cosmology, with a focus on the dark energy problem. Teleparallel Gravity is an alternative theory for gravitation was initiated by Albert Einstein, to base a unified theory of electromagnetism & gravity on the mathematical structure of distant parallelism which is equivalent to General Relativity (GR). However, it is conceptually different. For example, in GR geometry replaces the concept of force, & the trajectories are determined by geodesics. TG attributes gravitation to torsion, which accounts for gravitation by acting as a force. TG has already solved some old problems of gravitation (like the energy-momentum density of the gravitational field). The interest in TG has grown in the last few years.

Contents

1. Tetrads
2. Linear Connections & linear transformations
3. Orthogonal transformations
4. Connections revisited
5. Back to equivalence
6. Two gates into gravitation
7. Preliminaries; general concepts
8. Gauge transformations
9. Spacetime geometric structure
10. Lagrangian& field equations
11. Equivalence with General Relativity
12. Energy-momentum density of gravitation
13. Bianchi identities
14. Gravitational Lorentz force; torsion force equation
15. Curvature geodesic equation
16. Weak equivalence principle; equivalence of the action
17. Teleparallel spin connection
18. Teleparallel coupling prescription; application to fundamental field

Recommended Texts

1. Aldrovandi, R., & Pereira, J. G. (2013). *Teleparallel gravity* (1st ed.). Netherlands: Springer.
2. Stephani, H. (2004). *An introduction to special & general relativity* (3rd ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Rindler, W. (2007). *Relativity, special, & cosmology* (2nd ed.). Oxford: Oxford University Press.
2. Hartle, J. B. (2006). *Gravity, an introduction to Einstein's general relativity* (1sted.). Canada: Pearson Education.

MATH-8117 **Homotopy Theory** **3 (3+0)**

The course offers an introduction to algebraic topology centered around the theory of higher homotopy groups of a topological space. These groups offer more information than the homology or cohomology groups with which some students may be familiar, but are much harder to calculate. (In fact, the full computation of all the Homotopy groups of spheres is unknown & in some sense is the 'holy grail' of algebraic topology, although many special cases are known.) The course will start with a reminder about the fundamental group(oid), & the Homotopy relation on maps. Next, we'll define the (higher) homotopy groups of a space, & prove some basic properties about them. We'll discuss the action of the fundamental group on these groups, & the Serre long exact sequence of a fibration. This will enable us to perform some elementary calculations. We will discuss CW complexes & a proof of Whitehead's theorem about the construction of maps from the information about Homotopy groups. Further, we will use CW complexes to construct the Postnikov tower of a space, a context in which the famous Eilenberg-MacLane spaces come up. The Homotopy extensions & lifting property establishes an important relation between cofibrations & Serrefibrations (this is the motivation for one of Quillen's axioms for 'Homotopipcal algebra', axioms which play a dominant role in much of modern algebraic topology). We conclude the course by the Homotopy excision theorem & the Freudenthal suspension theorem, key results that lies at the basis of stable Homotopy theory.

Contents

1. Paths & path connected spaces
2. Homotopy of continuous mappings
3. Homotopy of paths
4. Homotopy classes
5. The fundamental group of a circle
6. Higher fundamental groups
7. The fundamental group of covering spaces
8. Torus
9. Orbit spaces
10. Punctured plane & surfaces

Recommended Texts

1. Paul, S. (2008). *Introduction to homotopy theory (Fields institute monographs)*. Rhode Island: American Mathematical Society.
2. Allen, H. (2001). *Algebraic topology*. Cambridge: Cambridge University Press.

Suggested Readings

1. Kosniowski, C. (1980). *A first course in algebraic topology*. Cambridge: Cambridge University Press.
2. Munkres, J. R. (2000). *Topology* (2nd ed.). New Jersey: Prentice Hall.
3. Whitehead, G.W. (1995). *Elements of homotopy theory*. New York: Springer.

MATH-8118 **Symmetries of Spacetimes** **3 (3+0)**

Like most courses of mathematical physics, overall objectives of this course are two-fold: acquire knowledge about physical aspects of the universe we live in & learn to quantitatively analyze physical problems. Spacetime symmetries are features of spacetime that can be described as exhibiting some form of symmetry. The role of symmetry in physics is important in simplifying solutions to many problems. Spacetime symmetries are used in the study of exact solutions of Einstein's field equations of general relativity. Spacetime symmetries are distinguished from internal symmetries. Physical problems are often investigated & solved by noticing features which have some form of symmetry. For example, in the Schwarzschild solution, the role of spherical symmetry is important in deriving the Schwarzschild solution & deducing the physical consequences of this symmetry (such as the nonexistence of gravitational radiation in a spherically pulsating star). In cosmological problems, symmetry plays a role in the cosmological principle, which restricts the type of universes that are consistent with large-scale observations. More specifically, students will acquire facility with special relativity & its application to relativistic particle dynamics. They will learn how to recognize various classes of elementary particles & predict the type of interactions responsible for their decays & scatterings. They will also practice performing order-of-magnitude estimates relevant for interpreting &/or judging the feasibility of a variety of modern physics experiments.

Contents

1. Spacetime symmetry
2. Geometry & matter
3. Spacetime & matter symmetry
4. Use of symmetries to simplify equations, physical significance of symmetries
5. Conservation laws corresponding to isometries
6. Homotheties & their significance
7. Collineation vectors, conformal symmetries
8. Killing vectors(KV's), curvature collineations (CC's)
9. Ricci collineations (RC's), mattercollinations (MC's)
10. Weylcollineations (WC's), relation between homotheties, KV's, CC's, RC's, MC's, WC's

Recommended Texts

1. Stephani, H. (2004). *Relativity, an introduction to special & general relativity* (3rd ed.). Cambridge: Cambridge University Press.
2. Rindler, W. (2007). *Relativity, special, & cosmology* (2nd ed.). Oxford: Oxford University Press.

Suggested Readings

1. Hartle, J. B. (2006). *Gravity: An introduction to Einstein's general relativity* (1st ed.). Canada: Pearson Education.

MATH-8119 Convex Analysis and Applications 3 (3+0)

The focus of this course to introduce the tools & recognition of convex optimization problems that arise in engineering. The basic theory of such problems, concentrating on results that are useful in computation, to give students a thorough underst&ing of how such problems are solved. Applications of Convex Analysis are found in almost all fields of pure & applied mathematics. Because of their various applications in areas such as the natural & engineering sciences as well as economics, new types of interesting inequalities are discovered every year. In the theory of differential equations, in the calculus of variations & ingeometry, fields which are dominated by inequalities, efforts are made to extend& improve the classical inequalities as the mathematical applications of convex functions.The study of convex analysis reflects the different aspects of modern mathematics. On one h&, there is the systematic search for the basic principles, such as the deeper underst&ing of monotonicity & convexity. On the other h&, findingthe solutions to an inequality requires often new ideas. Some of them have becomest&ard tools in mathematics. In view of the wide-ranging research related to convex analysis, several recent mathematical periodicals have been devoted exclusively to this topic.

Contents

1. Convex functions on the real line
2. Continuity & differentiability of convex functions
3. Characterizations, Differences of convex functions
4. Conjugate convex functions
5. Convex sets & affine sets
6. Convex functions on a normed linear space
7. Continuity of convex functions on normed linear space
8. Differentiable convex function on normed linear space
9. The support of convex functions, Differentiability of convex function on normed linear space

Recommended Texts

1. Niculescu, C. P., & Persson, L. E. (2018). *Convex functions & their applications, a contemporary approach.* New York: Springer.
2. Borwein, J. M.., & Lewis, A. S. (2010). *Convex analysis & nonlinear optimization, theory & examples.* New York: Springer.

Suggested Readings

1. Roberts, W., & Varberg, D. E. (1973). *Convex functions*. New York: Academic Press.
2. Rockafellar, R.T. (1970). *Convex analysis*. New Jersey: Princeton University Press.

MATH-8120 Numerical Solutions of Partial Differential Equations 3 (3+0)

This course addresses post graduate students of all fields who are interested in numerical methods for partial differential equations, with focus on a rigorous mathematical basis. Many modern & efficient approaches are presented, after fundamentals of numerical approximation are established. Of particular focus are a qualitative underst&ing of the considered partial differential equation, fundamentals of finite difference, finite volume, finite element, & spectral methods, & important concepts such as stability, convergence, & error analysis. Upon completion, students are able to solve following problems: (1) advection equation, heat equation, wave equation, Airy equation, convection-diffusion problems, KdV equation, hyperbolic conservation laws, Poisson equation, Stokes problem, Navier-Stokes equations, interface problems. (2) consistency, stability, convergence, Lax equivalence theorem, error analysis, Fourier approaches, staggered grids, shocks, front propagation, preconditioning, multi-grid, Krylov spaces, saddle point problems. (3) Finite differences, finite volumes, finite elements, ENO/WENO, spectral methods, projection approaches for incompressible owes, level set methods, particle methods, direct & iterative methods, multi-grid.

Contents

1. Boundary & initial conditions
2. Polynomial approximations in higher dimensions
3. Finite Element Method: The Galerkin method in one & more dimensions
4. Error bound on the Galarki method, the method of collocation
5. error bounds on the collocation method
6. Comparison of efficiency of the finite difference & finite element method
7. Finite Difference Method: Finite difference approximations
8. Application to solution of linear & non-linear partial differential equations appearing in physical problems

Recommended Texts

1. Silvia, B., Silvia, F., Giovanni, R., & Chi-Wang, S., (2008). *Numerical solutions of partial differential equations.* Switzerland: Birkhäuser Basel.
2. Johnson, C. (2009). *Numerical solutions of partial differential equations by the finite element method.* New York: Dover Publications.

Suggested Readings

1. Morton, K. W., & Mayers, D. F. (2005). *Numerical solution of partial differential equations.* Cambridge: Cambridge University Press.
2. Roe, P. L., & Chattot, J. J. (2002). *Innovative methods for numerical solutions of partial differential equations.* Singapore: World Scientific.
3. Jeffrey, A. (2002). *Applied partial differential equations: An introduction.* Cambridge: Academic Press.
4. Luis, A. C., François, G., Yan, G., Carlos, E. K., & Alexis, V. (2012). *Nonlinear partial differential equations.* Switzerland: Birkhäuser Basel.

MATH-8121 Representation Theory and The Symmetric Groups 3 (3+0)

The representation theory of the symmetric groups is a special case of the representation theory of finite groups. While representations over a field of characteristic zero are well-understood, fundamental questions over a field of prime characteristic remain widely open. The course will be algebraic & combinatorial in flavour. It will follow the approach taken by G. James. The final goal is to construct & parametrize the simple modules of the symmetric groups over an arbitrary field. On the way the theory over a field of characteristic zero will be developed. This course aims to give graduate students an introduction to the ordinary & the modular representation theory of symmetric groups. At the end of the course students should know Representation of a group by linear transformations of a vector space, Characters of representations of finite groups over the complex field & Irreducible representations of the symmetric group over the complex field.

Contents

1. Introduction to group representations
2. Matrix representation
3. The group algebra
4. Reducibility
5. Maschke's Theorem
6. Schur's Lemma
7. Group characters
8. Representations of the symmetric group (using Specht modules)
9. Combinatorial algorithms in representation theory
10. Robinson-Schensted-Knuth algorithm
11. Novelli-Pak-Stoyanovskii hook formula
12. Frobenius-Young determinantal formula
13. Schutzenberger'sjeu du taquin
14. Introduction to Symmetric functions
15. Schur functions
16. Littlewood-Richardson & Murnaghan-Nakayama Rules
17. Applications: Stanley's theory of differential posets
18. Fomin's concept of growths
19. Unimodality results
20. Stanley's symmetric function analogue of the chromatic polynomial of a graph

Recommended Texts

1. Bruce, E. S. (2001). The *symmetric group: representations, combinatorial algorithms, & symmetric functions*. New York: Springer.

Suggested Readings

1. William, F., & Young, T. (1997). *With applications to representation theory & geometry*. Cambridge: Cambridge University Press.
2. Jeffrey, A. (2002). *Applied partial differential equations: An introduction.* Cambridge: Academic Press.

MATH-8122 Non-Newtonian Fluid Mechanics 3 (3+0)

The objective of this course is to introduce & to illustrate the frequent & wide occurrence of non-Newtonian fluid behavior in a diverse range of applications, both in nature & in technology. Starting with the definition of a non-Newtonian fluid, different types of non-Newtonian characteristics are briefly described. Representative examples of materials (foams, suspensions, polymer solutions & melts), which, under appropriate circumstances, display shear-thinning, shear-thickening, visco-plastic, time-dependent & visco-elastic behavior will be presented. Each type of non-Newtonian fluid behavior has been illustrated via experimental data on real materials. This is followed by a short discussion on how to engineer non-Newtonian flow characteristics of a product for its satisfactory end use by manipulating its microstructure by controlling physico-chemical aspects of the system. Finally, we touch upon the ultimate question about the role of non-Newtonian characteristics on the analysis & modeling of the processes of pragmatic engineering significance. To consider the motion of the particles, the convection–diffusion equation will be used & perform a parametric study (after having non-dimensionalized the equations). The results for concentration, velocity, & temperature profiles in terms of various dimensionless numbers will also be discussed.

Contents

1. Classification of non-Newtonian fluids
2. Rheological formulae (time-independent fluids)
3. Thixotropic fluids & viscoelastic fluids
4. Variable viscosity fluids, cross viscosity fluids
5. The deformation rate, viscoelastic equation, materials with short memories
6. Time dependent viscosity, the Rivlin-Ericksen fluid
7. Basic equations of motion in rheological models
8. The linear viscoelastic liquid, Couette flow, Poiseuille flows
9. The current semi-infinite field,axial oscillatory tube flow
10. Angular oscillatory motion, periodic transients
11. Basic equations in boundary layer theory, orders of magnitude
12. Truncated solutions for viscoelastic flow, similarity solutions, turbulent boundary layers, stability analysis

Recommended Texts

1. Glowinski, R., Jinchao, X., & Philippe, G. C. (2011). *Numerical methods for non-newtonian fluids*. The Netherlands: North Holland.
2. Clayton, T. C., Donald, F. E., John, A. R., & Barbara, C. W. (2008). *Engineering fluid mechanics*. New York: Wiley.

Suggested Readings

1. Bohme, G. (2012). *Non-newtonian fluid mechanics*. The Netherlands: North Holland Publisher.
2. Bird, R. B., Armstrong, R. C., & Hassager, O. (1987). *Dynamics of polymeric liquids*. New York: John Wiley & Sons.

MATH-8123 Orthogonal Polynomials 3 (3+0)

The main purpose of this course is Orthogonal Polynomials & Special Functions (OP&SF). In this field, together with its applications, there are several strong research groups located at various universities & institutes in the European Union. In order to keep up the strength of these research groups we need to bring young researchers to an excellent research level, enhance international cooperation between research groups, especially at the level of young researchers, enhance the functioning of European network in OP&SF, get young researchers interested in OP&SF. In mathematics, an orthogonal polynomial sequence is a family of polynomials such that any two different polynomials in the sequence are orthogonal to each other under some inner product. The most widely used orthogonal polynomials are the classical orthogonal polynomials, consisting of the Hermite polynomials, the Laguerre polynomials & the Jacobi polynomials together with their special cases the Gegenbauer polynomials, the Chebyshev polynomials, the Legendre polynomials, Ultraspherical polynomials & other polynomial sets.

Contents

1. Generating functions
2. The classical orthogonal polynomials
3. Normalization
4. Pure recurrence relations
5. Differential recurrence relations
6. Bessel polynomials
7. Legendre polynomials
8. Hermit polynomials
9. Laguerre polynomials
10. Jacobi polynomials
11. Chebyshev polynomials
12. Gegenbauer polynomials
13. Ultraspherical polynomials
14. Euler polynomials
15. Other polynomial sets

Recommended Texts

1. Richard, B. (2016). *Special functions & orthogonal polynomials*. Cambridge: Cambridge University Press.
2. Rainville, E. D. (1971). *Special functions* (3rd ed.). New York: The Macmillan Company.

Suggested Readings

1. Andrews, G. E., Richard, A., & Roy, R. (2000). *Special functions.* Cambridge: Cambridge University Press.
2. Mathai, A. M., & Houbold, H. J. (2008). *Special functions for applied scientists*. New York: Springer Science & Business Media, LLC.
3. Singh, U. P., & Denis, R. Y. (2001). *Special functions & their applications.* India: Dominant Publishers & Distributors.

MATH-8124 Numerical Spline Techniques-I 3 (3+0)

In the mathematical field of numerical analysis, spline interpolation is a form of interpolation where the interpolant is a special type of piecewise polynomial called a spline. Spline interpolation is often preferred over polynomial interpolation because the interpolation error can be made small even when using low degree polynomials for the spline. Spline interpolation avoids the problem of Runge's phenomenon, in which oscillation can occur between points when interpolating using high degree polynomials. This course is designed to teach students about splines. The course aims at inculcating in the students to apply various techniques used in the theory of splines, underst& & do calculations about spline functions the collocation techniques based on spline functions. Upon completing this course the students should have a theoretical background of theory of splines & should be able to solve a variety of ordinary & partial differential equations using spline collocation techniques.

Contents

1. B-spline representation in terms of divided differences
2. The B-spline representation of spline functions
3. Computational considerations, the representation of B-splines (Method based on the recursive definition of divided differences)
4. Classification of linear/non-linear second orders PDEs in two & more variables. Laplacian Equations
5. Heat Equation. Review for the representation of B-splines (Polynomial B-spline, Extended B-spline, exponential B-spline
6. Trigonometric B-spline (Method based on the recursive definition of divided differences)
7. B-spline numerical methods of solving Laplace, Heat & Wave equations. Finite difference & finite element methods
8. Error analysis of methods. Trigonometric/Exponential/Extended B-spline Numerical Solutions for Elliptic, Parabolic & Hyperbolic PDEs
9. B-spline approach for solving system of linear/non-linear PDEs, Irregular Boundaries. Error & stability analysis of B-spline methods

Recommended Texts

1. De-Boor, C. (2001). A *practical guide to spline.* New York: Springer Verlag.
2. Farin, G. (2002). *Curves & surfaces for computer aided geometric design: a practical guide.* Cambridge: Academic Press Inc.
3. Shazalina, B. M. (2016). Z.*B-splines collocation approach for solving partial differential equations.* Malaysia: Ph.D. Thesis, USM.

Suggested Readings

1. Prenter, M. (1989). *Splines & variational methods*. New York: John Wiley & Sons.
2. Bartels, R. H., Bealty, J. C., & Beatty, J. C. (2006). *An introduction to spline for use in computer graphics & geometric modeling*. The Netherlands: Morgan Kaufmann Publisher.
3. Wang, R. H. (2005). *Multivariate spline functions & their applications (mathematics & its applications).* Switzerland: Science Press/ Kluwer Academic Publishers.

MATH-8125 **Numerical Spline Techniques-II** **3 (3+0)**

In mathematics, a spline is special function defined piecewise by polynomials. In interpolating problems, spline interpolation is often preferred to polynomial interpolation because it yields similar results, even when using low degree polynomials. In the computer science subfields of computer-aided design & computer graphics, the term spline more frequently refers to a piecewise polynomial (parametric) curve. Splines are popular curves in these subfields because of the simplicity of their construction, their ease & accuracy of evaluation, & their capacity to approximate complex shapes through interactive curve design. This course is designed to teach students about basics of scientific computing for solving differential equations using B-spline collocation techniques & learn how to match numerical method to mathematical properties. This course gives the students the knowledge of problem classes, basic mathematical & numerical concepts & software for solution of engineering & scientific problems formulated as differential equations. After successful completion, students should be able to design, implement & use numerical methods for computer solution of scientific problems involving differential equations. B-spline collocation method is applied on problems which are not discussed in the course using MATLAB/Mathematica.

Contents

1. Classification of linear/non-linear second orders ODEs. Discuss the types of boundary conditions,
2. Theory & implementation of numerical methods for initial & boundary value problems in ordinary differential equations
3. One-step, linear multi-step, Runge-Kutta, & Extrapolation methods; convergence
4. stability, B-spline collocation methods for numerical solution of initial & boundary values problems
5. B-spline for solving class of singular & non-singular problems
6. Trigonometric B-spline approach for solving second order system of linear/non-linear ODEs
7. B-spline solution for nonlinear differential equation arising in general relativity
8. Bratu'sproblem, Perturbation's problem

Recommended Texts

1. De-Boor, C. (2001). A *practical guide to spline.* New York: Springer Verlag.
2. Farin, G. (2002). *Curves & surfaces for computer aided geometric design: a practical guide.* Cambridge: Academic Press Inc.
3. Shazalina, B. M. (2016). Z.*B-splines collocation approach for solving partial differential equations.* Malaysia: Ph.D. Thesis, USM.

Suggested Readings

1. Prenter, M. (1989). *Splines & variational methods*. New York: John Wiley & Sons.
2. Bartels, R. H., Bealty, J. C., & Beatty, J. C. (2006). *An introduction to spline for use in computer graphics & geometric modeling*. The Netherlands: Morgan Kaufmann Publisher.
3. Wang, R. H. (2005). *Multivariate spline functions & their applications (mathematics & its applications).* Switzerland: Science Press/ Kluwer Academic Publishers.

MATH-8126 **Computational Geometry** **3 (3+0)**

The objectives of this course are as follows, introduce rigorous algorithmic analysis for problems in Computational Geometry, discuss applications of Computational Geometry to graphical rendering, introduce the notions of Voronoi diagrams & Delaunay Triangulations, & develop expected case analyses for linear programming problems in small dimensions. This course also provides an introduction to the key concepts, problems, techniques & data structures within computational geometry. Upon successful completion of this course, students will be able to analyze r&omized algorithms for small domain problems, use line-point duality to develop efficient algorithms, apply geometric techniques to real-world problems in graphics & solve linear programs geometrically. Other important applications of computational geometry include robotics (motion planning & visibility problems), geographic information systems (GIS) (geometrical location & search, route planning), integrated circuit design (IC geometry design & verification), computer-aided engineering (CAE) (mesh generation), computer vision (3D reconstruction).

Contents

1. Introduction: polygon trapezoidation
2. Geometry of convex hull
3. Voronoi diagram
4. Delaunay triangulation
5. Algorithms for their construction
6. Arrangements of curves, surfaces & lines
7. Theory of oct-trees
8. Kd-trees
9. BSP trees
10. Applications
11. Special topics

Recommended Texts

1. Berg, M. de., Kreveld, M. van., Overmars, M., & Schwarzkopf, O. (1997). *Computational geometry: algorithms &* applications (1st ed.). New York: Springer-Verlag.
2. Boissonnat, J. D., & Teillaud, M. (2007). *Effective computational geometry for curves & surfaces.* New York: Springer.

Suggested Readings

1. Sack, J. R., & Urrutia, J. (2000). *Handbook of computational geometry.* The Netherlands: Elsevier.
2. Clara, I. G., & Alberto, M. (2001). *Computational geometry on surfaces: performing computational geometry on the cylinder, the sphere, the torus, & the cone.* New York: Springer.

MATH-8127 Special Functions and Statistical Distribution 3 (3+0)

The course is aimed at providing students with a formal treatment of probability theory, equipping students with essential tools for statistical analyses at the graduate level & Fostering underst&ing through real-world statistical applications. The probability distributions are used extensively in solving problems both in probability & statistical inference. The gamma & beta distributions derive their names from the well known gamma & beta functions which are important in many areas of mathematics & probability theory of statistics. A probability distribution is a mathematical function that provides the probabilities of the occurrence of various possible outcomes in an experiment. Probability distributions are used to define different types of r&om variables in order to make decisions based on these models. There are two types of r&om variables: discrete & continuous. Depending on what category the r&om variable fits into, a statistician may decide to calculate the mean, median, variance, probability, or other statistical calculations using a different equation associated with that type of r&om variable. This is important because, as experiments may become more complicated, the st&ard formulas that are used to calculate these parameters (like the mean) will no longer produce accurate results.

Contents

1. Basic concepts
2. Continuous & discrete distribution
3. Skewness & kurtosis
4. Moments of continuous & discrete distribution
5. Probability & probability distribution
6. Expected values, r&om variable, binomial distribution, uniform distribution
7. Moments generating function
8. Cummulants & Cumulative function, exponential distribution
9. Poisson distribution
10. Approximation to the binomial
11. Hypergeometric distribution, Geometric distribution, Normal distribution
12. Gamma function & Gamma distribution
13. Beta function& beta distributions of first & second kind
14. Chi square distribution
15. T-distribution, F distribution, k-distributions

Recommended Texts

1. Walac, C. (2007). A *handbook on statistical distributions for experimentalists, particle physics group fysikum*. Sweden: University of Stockholm.
2. Ronald, E. W., Raymond, H. M., Sharon, L. M., & Keying, E. (2007). *Probability & statistics for engineers & scientist.* Canada: Pearson Prentice Hall.

Suggested Readings

1. Hasting, N. A. J., & Peacock, J. B. (1975). *Statistical distributions*. Oxford: Butterworth & Company Ltd.
2. Rainville, E. D. (1971). *Special functions*. New York: Macmillan Company Press.

MATH-8128 **Fuzzy Analysis** **3 (3+0)**

The only way a computer can process information like a human is for the control systems to incorporate fuzzy logic into its decision-making activities. This course will review the creation of fuzzy logic (1960's) and lay the foundation for how fuzzy logic comes about and how fuzzy sets help to create better control decisions and control systems. Defined in the course is the idea of crisp data versus fuzzy data, fuzzy data sets and how one can "defuzzify" data to create clear (crisp) decisions. As an example, picture a driver entering a medium-busy interstate highway. The course will introduce the idea that this driver does make life and death decisions without ever knowing 'true data' such as how fast they may be going, how far away other drivers might be or how fast anyone else is traveling. It is this "human processing" of information which we explore through the world of "fuzzy logic". Fuzzy Logic is a decision-making system. It deals with vague & imprecise information. Fuzzy sets & fuzzy logic gives us one way of representing this uncertainty & reasoning with them.

Contents

1. Fuzzy Sets
2. Level sets
3. Special types of fuzzy sets
4. Zadeh's extension principle
5. Fuzzy functions
6. Sup-min extension principle
7. Interval arithmetic
8. Fuzzy numbers & fuzzy arithmetic
9. Fuzzy metric spaces
10. Fuzzy metric spaces
11. Inner product, Support function
12. Embedding results
13. Continuous fuzzy functions
14. Measurable fuzzy functions
15. Integrable fuzzy functions
16. Differentiable fuzzy functions

Recommended Texts

1. Barnabas, B. (2013). *Mathematics of fuzzy sets & fuzzy logic.* New York: Springer.
2. Choudary, A. D. R., & Lupulescu, V. (2010). *Fuzzy sets & fuzzy differential equations*. Lahore: Abdus Salam SMS.

Suggested Readings

1. Lakshmikatham, V., Bhaskar, T. G., & Devi, J. V. (2006). *Theory of set differential equations in metric spaces, Cambridge scientic.* Cambridge: Cambridge.
2. Lakshmikantham, V., & Mohapatra, R. (2003). *Theory of fuzzy differential equations & inclusions*. London: Taylor & Francis.
3. Negoita, C., & Ralescu, D. (1975). *Applications of fuzzy sets to systems analysis*. New York: Wiley.

MATH-8129 **Advanced Partial Differential Equation** **3 (3+0)**

The course objectives are to learn the basics analytical methods to solve partial differential equations (PDE). Partial Differential Equations (PDEs) are at the heart of applied mathematics & many other scientific disciplines. Advanced Partial Differential Equations are at the heart of applied mathematics & many other scientific disciplines. In fact, the most important applications of advanced PDEs arise in finding the solutions of boundary value problems. The focus of the course is the concepts & techniques for solving the advanced partial differential equations (PDE) that permeate various scientific disciplines. Applications include problems from fluid dynamics, electrical & mechanical engineering, materials science & quantum mechanics. Advanced Partial Differential Equations (PDEs) occur frequently in many areas of mathematics. This is highly important to extend earlier work on PDEs by presenting a variety of more advanced solution techniques together with some of the underlying theories. In fact, the most important applications arise in finding the solutions of boundary value problems in the theory of partial differential equations of multi orders for boundary value problems in equations of elliptic, parabolic & hyperbolic differential equations.

Contents

1. Classification of Partial Differential Equations
2. Canonical form; Laplace
3. Wave & Diffusion Equations
4. Partial Differential Equations with at least 3 independent variables
5. Nonhomogeneous problems
6. Green function for time independent problems
7. Infinite domain problems
8. Green function for time dependent problems
9. Wave equation & the method of characteristics
10. Mathematical modeling

Recommended Texts

1. Silvia, B., Silvia, F., Giovanni, R., & Chi-Wang, S. (2008). *Numerical solutions of partial differential equations.* The Netherlands: Birkhäuser Basel.
2. Claes, J. (2009). *Numerical solutions of partial differential equations by the finite element method.* New York: Dover Publications.

Suggested Readings

1. Roe, P. L., & Chattot, J. J. (2002). *Innovative methods for numerical solutions of partial differential equations.* Singapore: World Scientific.
2. Alan, J. (2002). *Applied partial differential equations: An introduction*. Cambridge: Academic Press.
3. Luis, A. C., François, G., Yan, G., Carlos, E. K., & Alexis, V. (2012). *Nonlinear partial differential equations.* The Netherlands: Birkhäuser Basel.
4. Recent published research papers.